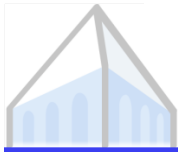


# Natural Language Processing



Large Language Models

# Language Modeling



# Recap: What is a language model?

---

- Language models assign a probability to a sequence of words

$$p(\bar{y})$$

- We can decompose this probability using the chain rule

$$p(\bar{y}) = \prod_{i=1}^T p(y_i | y_{0:i-1})$$

- We can autoregressively generate sequences from the language model by sampling from its token-level probability

$$p(y_i | y_{0:i-1})$$

- We can condition on our language distribution on something else

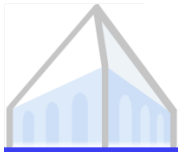
$$p(y_i | y_{0:i-1}; \bar{x})$$



# What can we do with language models?

---

- Computing probabilities of a sequence
- Autoregressive sequence generation

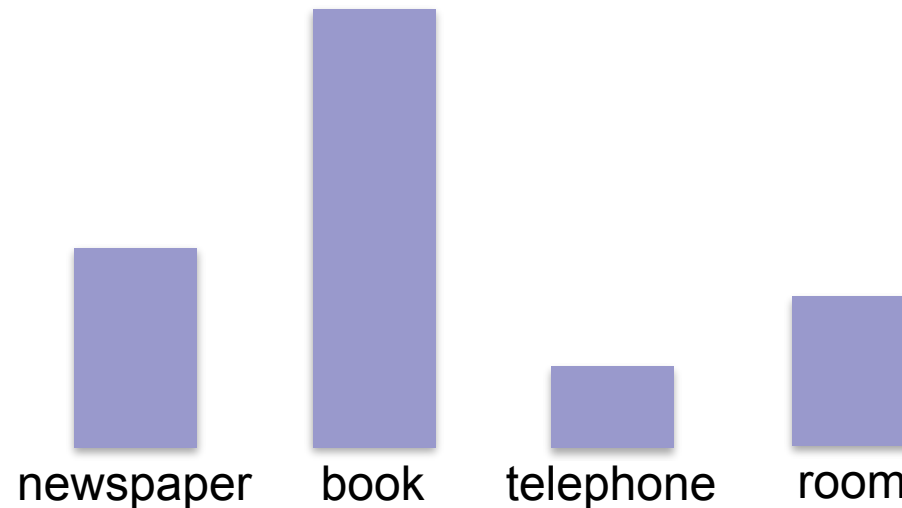


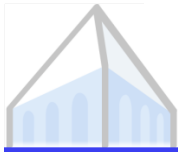
# Decoding strategies

---

- Argmax (greedy decoding)

$$y_T = \arg \max_{y \in \mathcal{V}} p(y \mid y_{0:t-1})$$



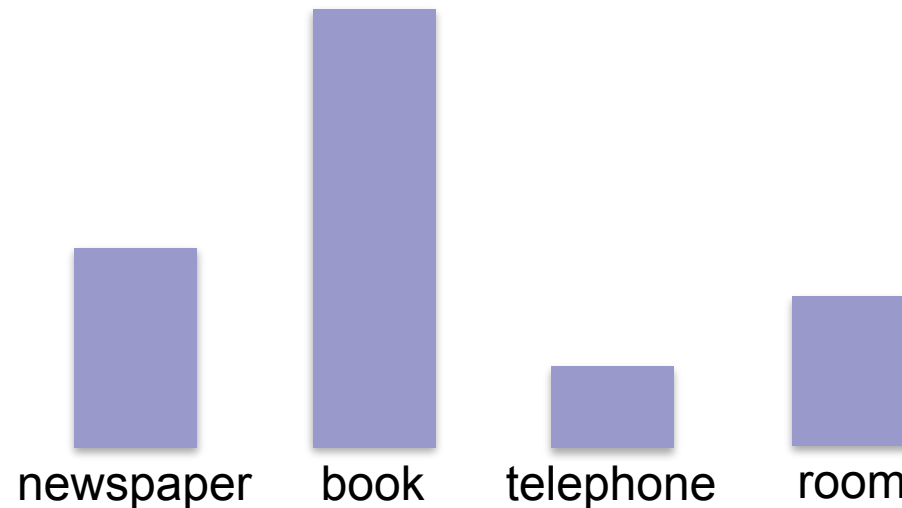


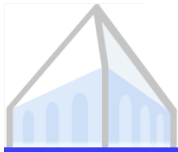
# Decoding strategies

- Argmax (greedy decoding)
- Sampling from language model directly

$$y_T = \arg \max_{y \in \mathcal{V}} p(y \mid y_{0:t-1})$$

$$y_T \sim p(\cdot \mid y_{0:t-1})$$





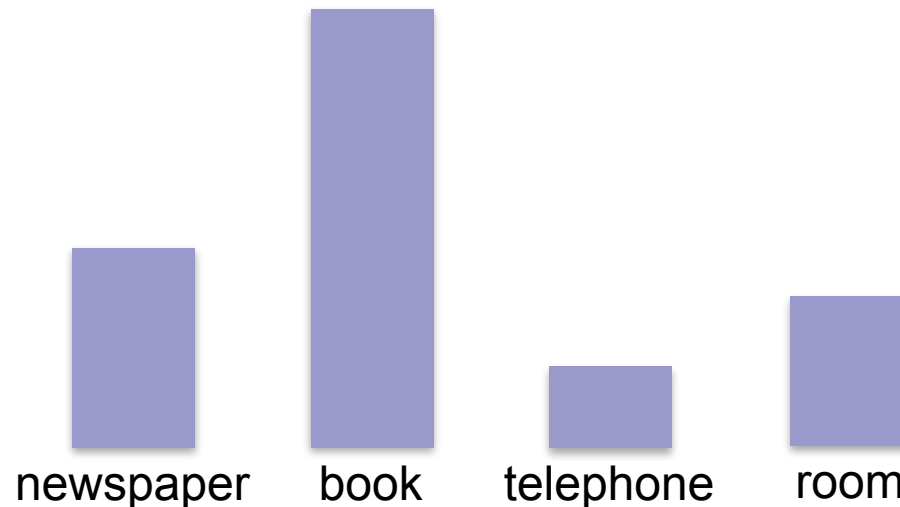
# Decoding strategies

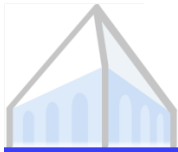
- Argmax (greedy decoding)
- Sampling from language model directly
- Adjusting temperature of distribution

$$y_T = \arg \max_{y \in \mathcal{V}} p(y \mid y_{0:t-1})$$

$$y_T \sim p(\cdot \mid y_{0:t-1})$$

$$p'(y_T = y) = \frac{\exp(z_y/T)}{\sum_{y' \in \mathcal{V}} \exp(z_{y'}/T)}$$





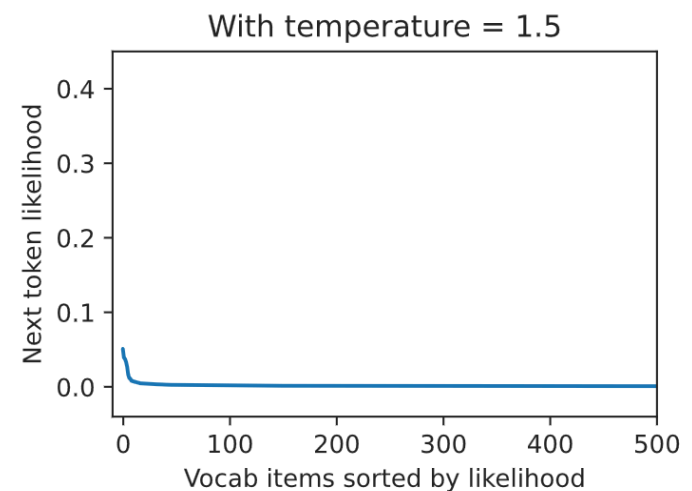
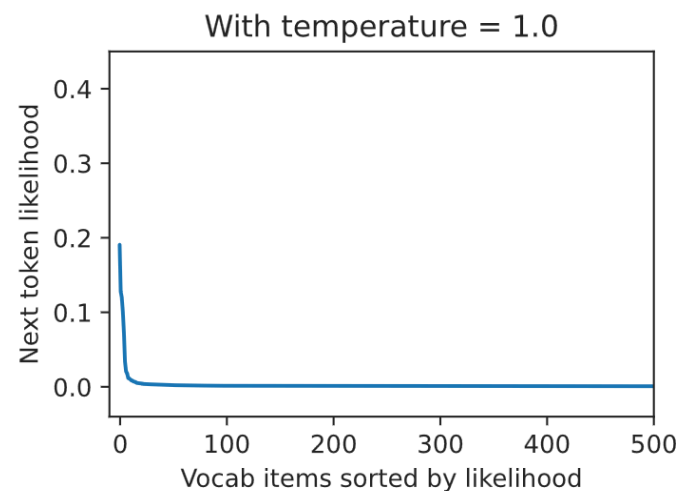
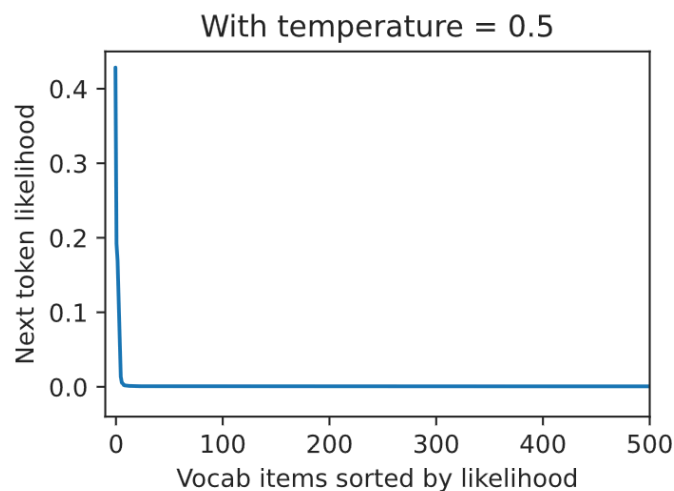
# Decoding strategies

- Argmax (greedy decoding)
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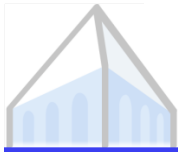
$$y_T = \arg \max_{y \in \mathcal{V}} p(y \mid y_{0:t-1})$$

$$y_T \sim p(\cdot \mid y_{0:t-1})$$

$$p'(y_T = y) = \frac{\exp(z_y/T)}{\sum_{y' \in \mathcal{V}} \exp(z_{y'}/T)}$$

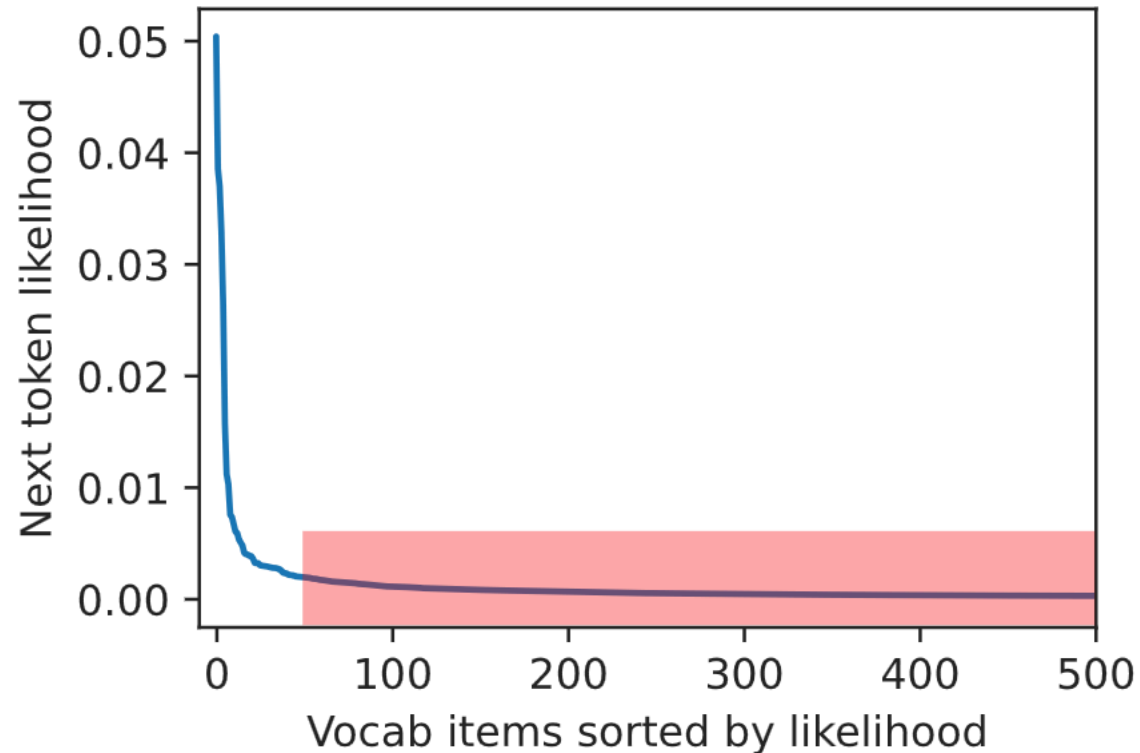


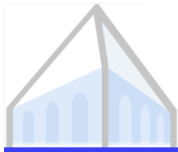




# Decoding strategies

- Top-k sampling: reassign probability mass from all but the top  $k$  tokens to the top  $k$  tokens





# Decoding strategies

- Nucleus sampling: reassign probability mass to the most probable tokens whose cumulative probability is at least  $p$

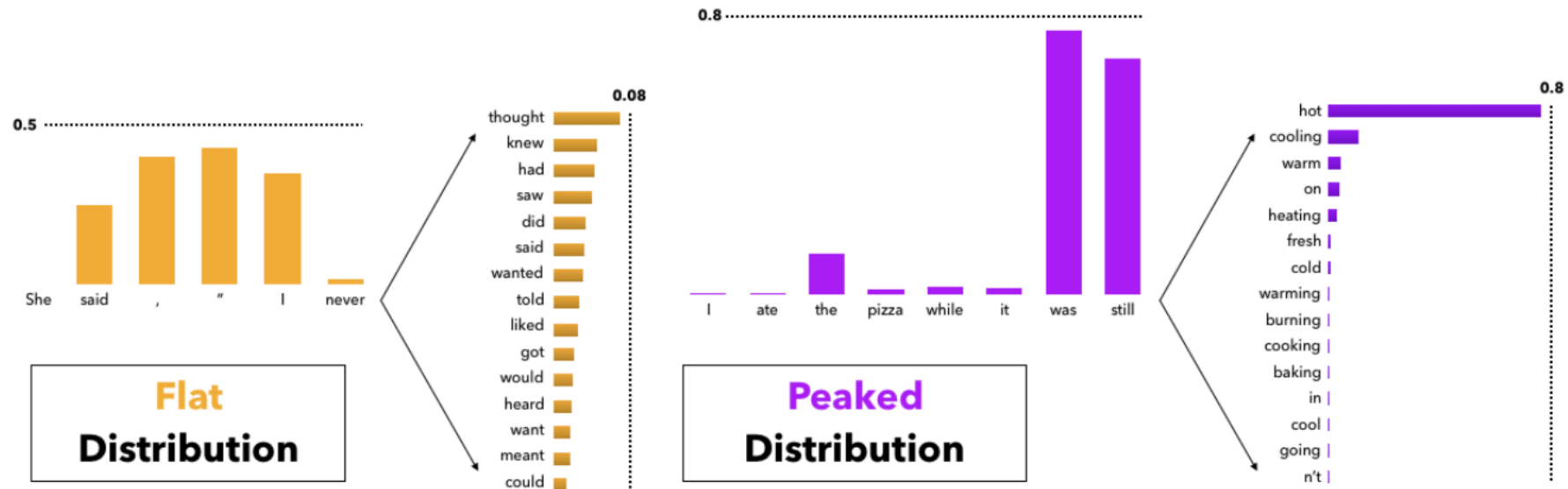
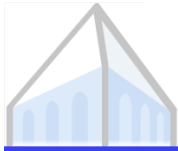


Figure 5: The probability mass assigned to partial human sentences. Flat distributions lead to many moderately probable tokens, while peaked distributions concentrate most probability mass into just a few tokens. The presence of flat distributions makes the use of a small  $k$  in top- $k$  sampling problematic, while the presence of peaked distributions makes large  $k$ 's problematic.



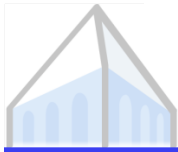
# Beam search

---

- It's intractable to find the *most probable sequence* according to a language model
- Greedy search doesn't yield the most probably sequence
- Instead: beam search
  - Approximate the search by keeping around candidate continuations
  - At the end, choose the highest probability sequence in the beam

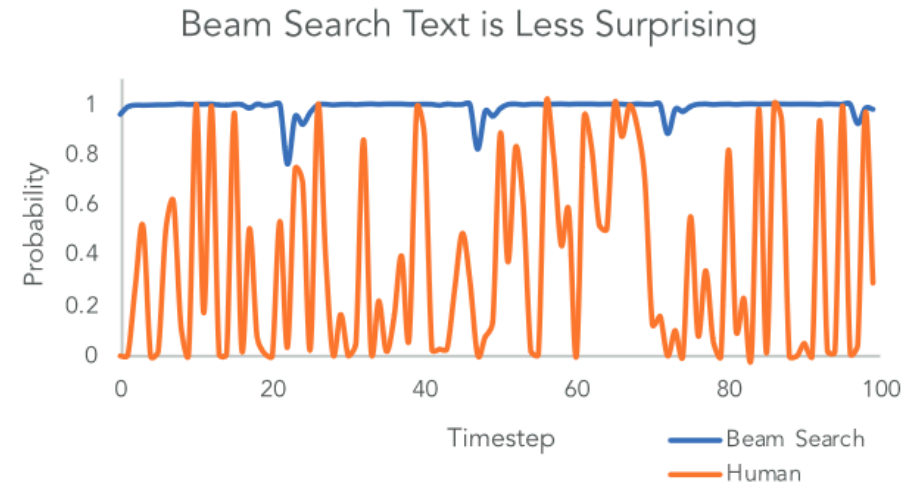
$$\bar{y}^* = \arg \max_{\bar{y} \in \mathcal{V}^*} p(\bar{y})$$

$$y_t = \arg \max_{y \in \mathcal{V}} p(y \mid y_{0:t-1})$$



# Beam search

- But do we even want to find the highest-probability sequence according to a LM?
- Human language is noisy and surprising
- Optimizing for LM probability leads to repetitive and uninteresting text

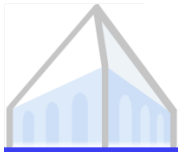


## Beam Search

...to provide an overview of the current state-of-the-art in the field of computer vision and machine learning, and to provide an overview of the current state-of-the-art in the field of computer vision and machine learning, and to provide an overview of the current state-of-the-art in the field of computer vision and machine learning, and to provide an overview of the current state-of-the-art in the field of computer vision and machine learning, and...

## Human

...which grant increased life span and three years warranty. The Antec HCG series consists of five models with capacities spanning from 400W to 900W. Here we should note that we have already tested the HCG-620 in a previous review and were quite satisfied With its performance. In today's review we will rigorously test the Antec HCG-520, which as its model number implies, has 520W capacity and contrary to Antec's strong beliefs in multi-rail PSUs is equipped...



# Beam search

- But do we even want to find the highest-probability sequence according to a LM?
- Human language is noisy and surprising
- Optimizing for LM probability leads to repetitive and uninteresting text

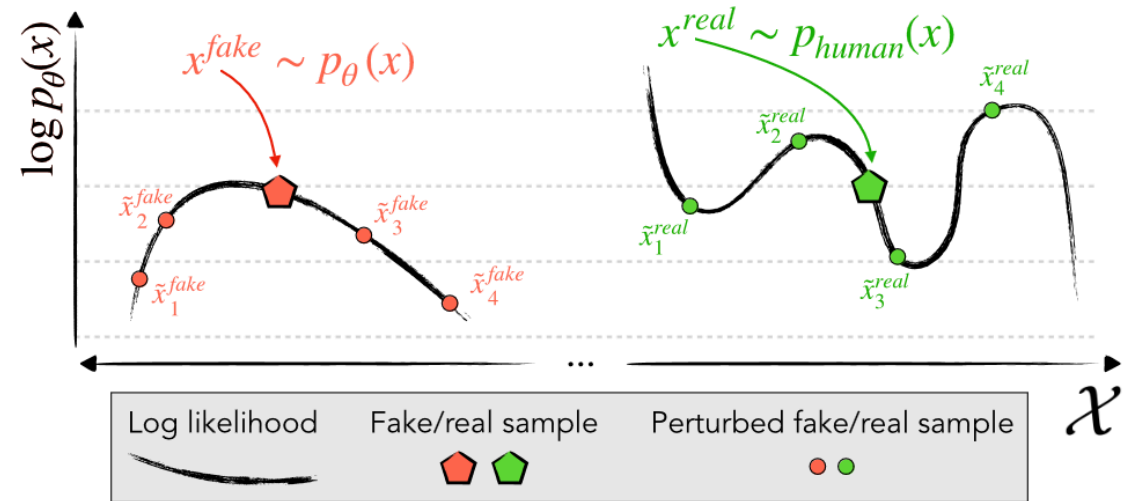
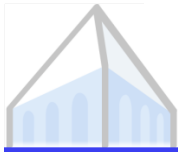


Figure 2. We identify and exploit the tendency of machine-generated passages  $x \sim p_\theta(\cdot)$  (left) to lie in negative curvature regions of  $\log p(x)$ , where nearby samples have lower model log probability on average. In contrast, human-written text  $x \sim p_{\text{real}}(\cdot)$  (right) tends not to occupy regions with clear negative log probability curvature; nearby samples may have higher or lower log probability.



# Recap: Feedforward Networks

- Tokenize
- Embed
- Concatenate
- Linear layer
- Softmax
- Fixed window?
- Word averaging?

output distribution

$$\hat{y} = \text{softmax}(U\mathbf{h} + \mathbf{b}_2) \in \mathbb{R}^{|V|}$$

hidden layer

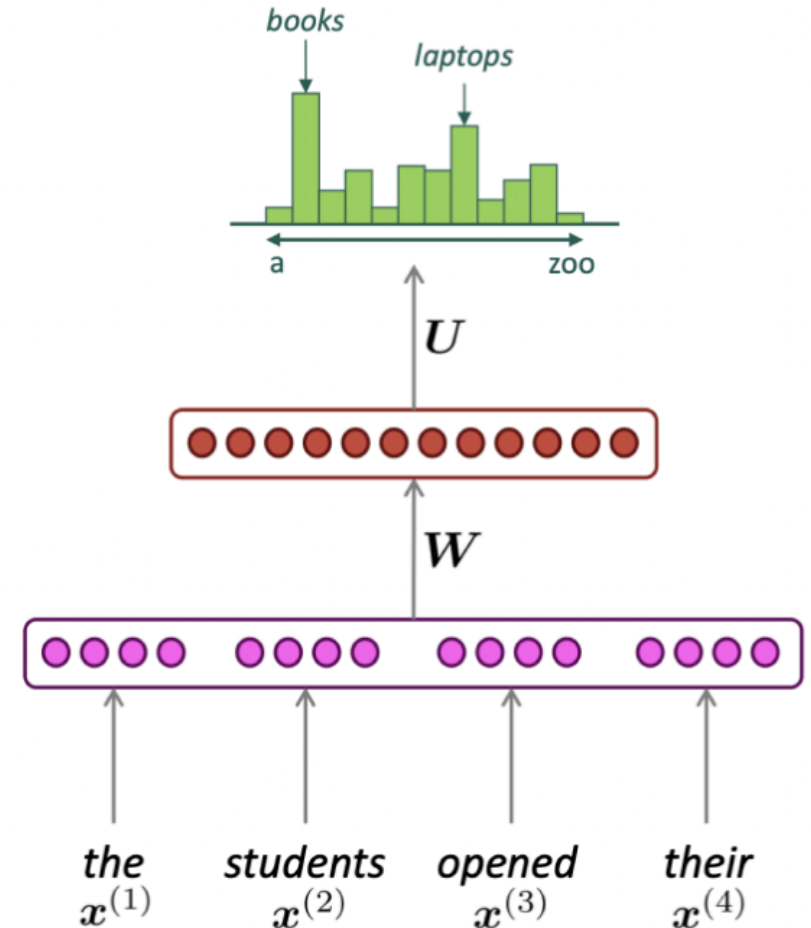
$$\mathbf{h} = f(\mathbf{W}\mathbf{e} + \mathbf{b}_1)$$

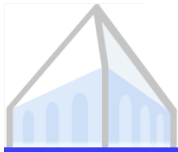
concatenated word embeddings

$$\mathbf{e} = [e^{(1)}; e^{(2)}; e^{(3)}; e^{(4)}]$$

words / one-hot vectors

$$\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \mathbf{x}^{(3)}, \mathbf{x}^{(4)}$$





# Recap: Recurrence

output distribution

$$\hat{y}^{(t)} = \text{softmax}(U\mathbf{h}^{(t)} + \mathbf{b}_2) \in \mathbb{R}^{|V|}$$

hidden states

$$\mathbf{h}^{(t)} = \sigma(W_h \mathbf{h}^{(t-1)} + W_e \mathbf{e}^{(t)} + \mathbf{b}_1)$$

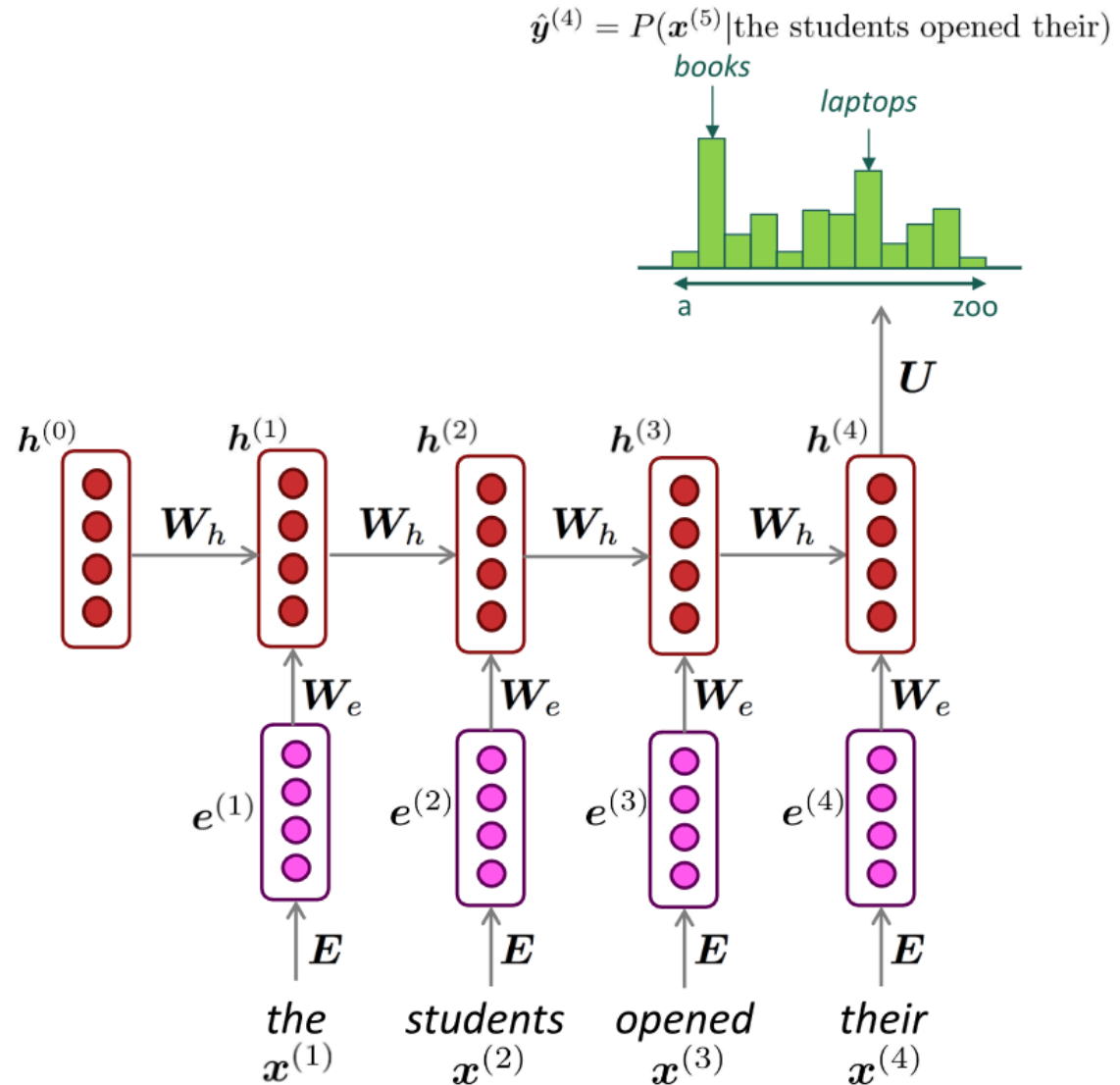
$\mathbf{h}^{(0)}$  is the initial hidden state

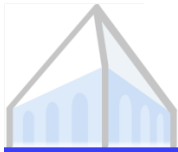
word embeddings

$$\mathbf{e}^{(t)} = E\mathbf{x}^{(t)}$$

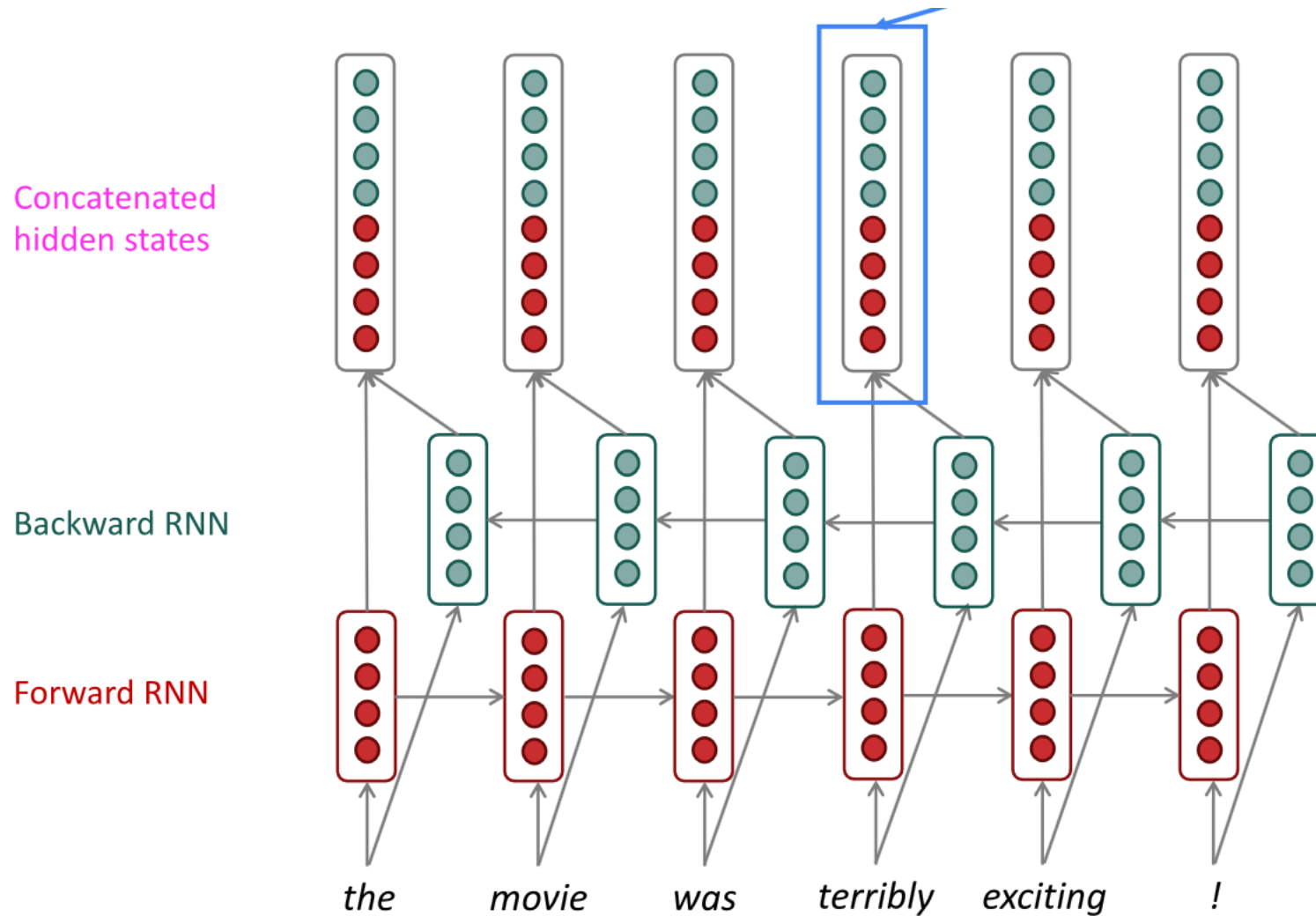
words / one-hot vectors

$$\mathbf{x}^{(t)} \in \mathbb{R}^{|V|}$$

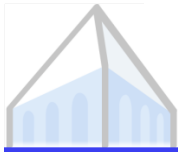




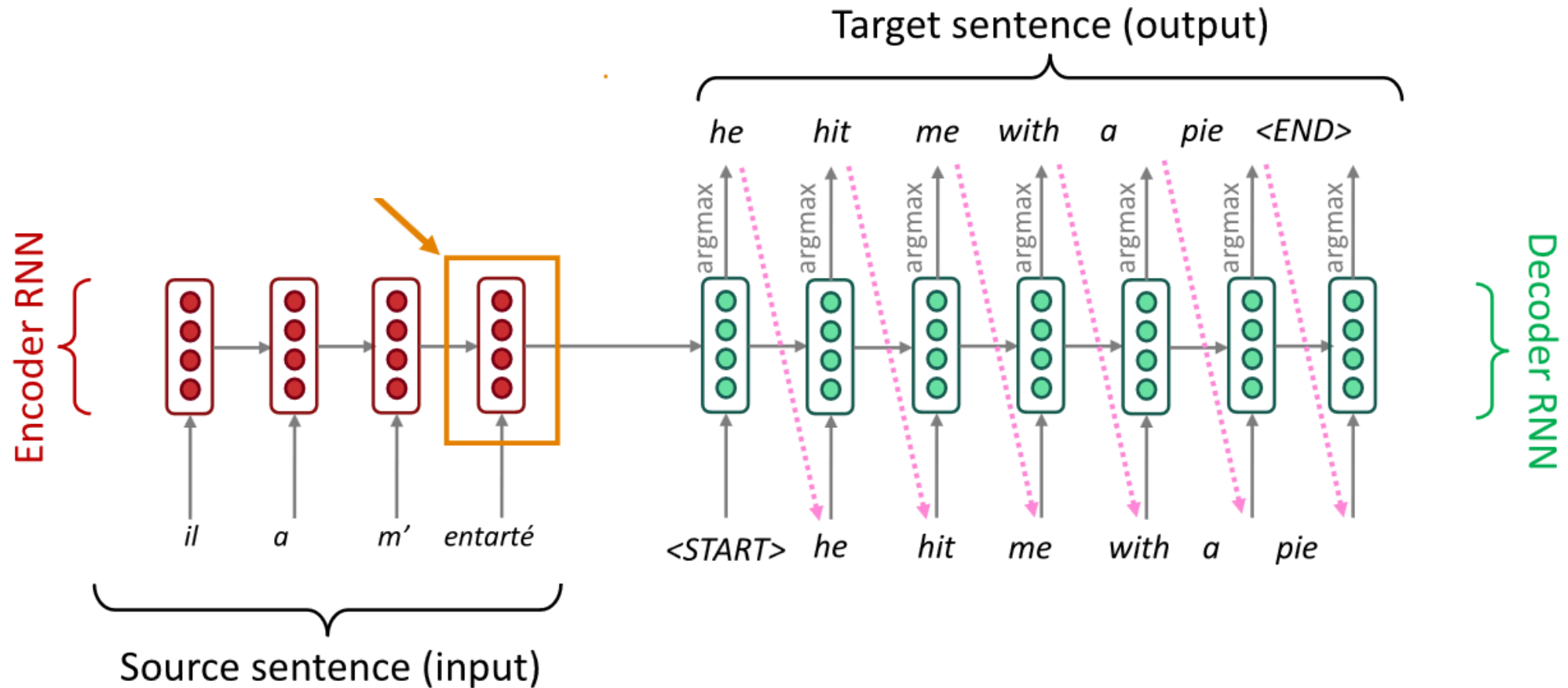
# Recap: Recurrence

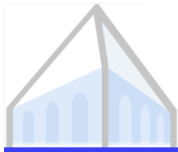




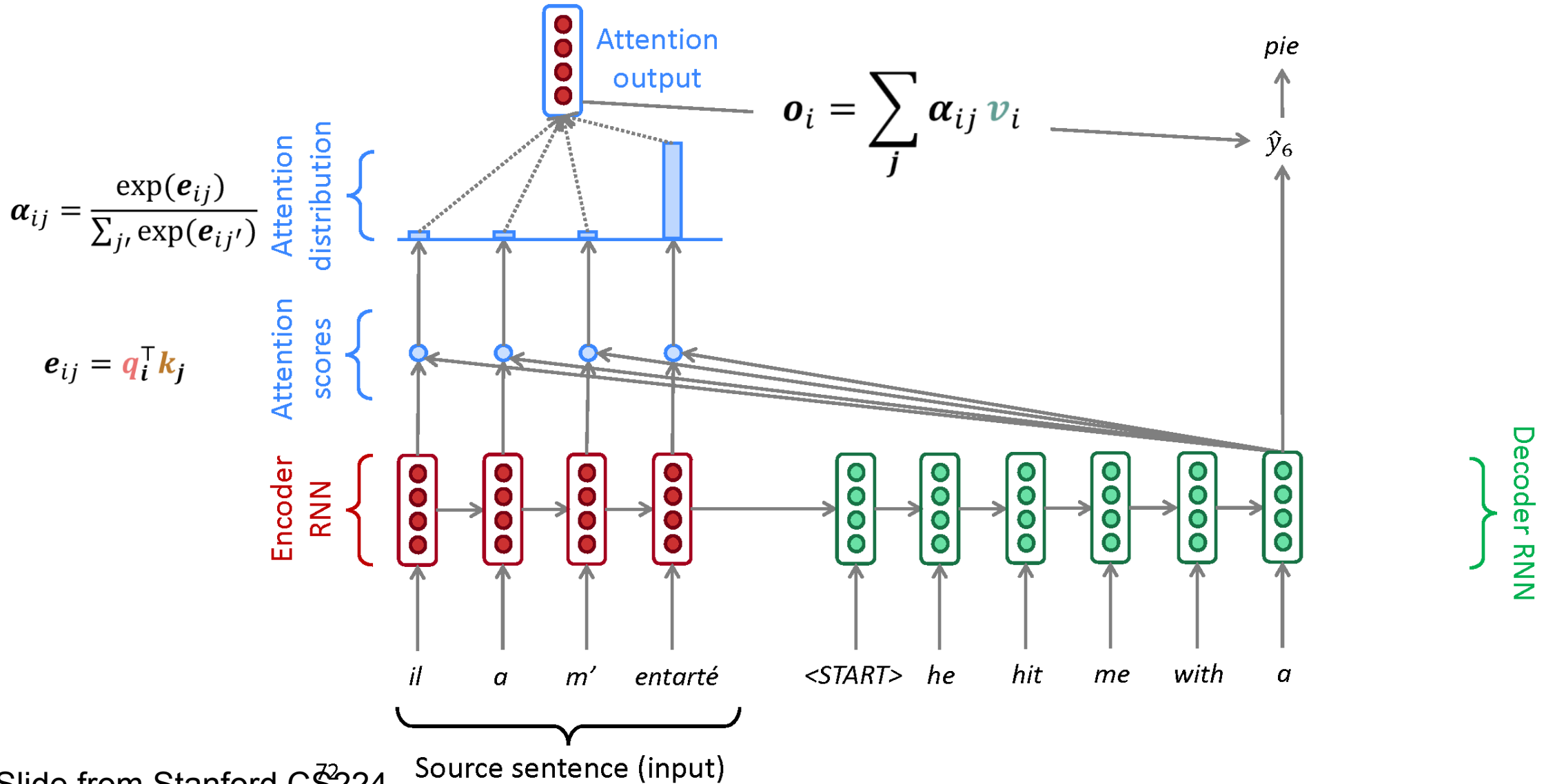


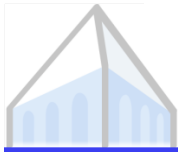
# Recap: Recurrence





# Recap: Attention





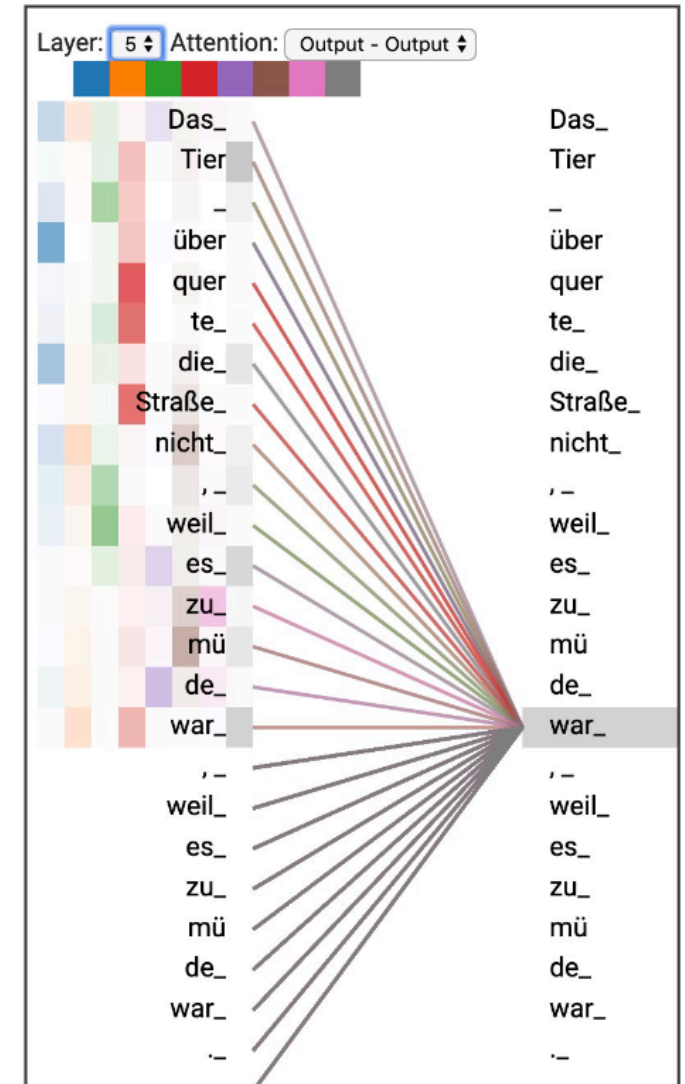
# Recap: Attention

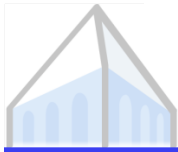
- Generic dot-product attention:

$$e_{ij} = \mathbf{q}_i^\top \mathbf{k}_j \quad \alpha_{ij} = \frac{\exp(e_{ij})}{\sum_{j'} \exp(e_{ij'})} \quad \mathbf{o}_i = \sum_j \alpha_{ij} \mathbf{v}_j$$

- Self-attention: queries, keys, and values are all different transformations of the same item-level representation of some sequence:

$$\mathbf{q}_i = Q \mathbf{x}_i \text{ (queries)} \quad \mathbf{k}_i = K \mathbf{x}_i \text{ (keys)} \quad \mathbf{v}_i = V \mathbf{x}_i \text{ (values)}$$



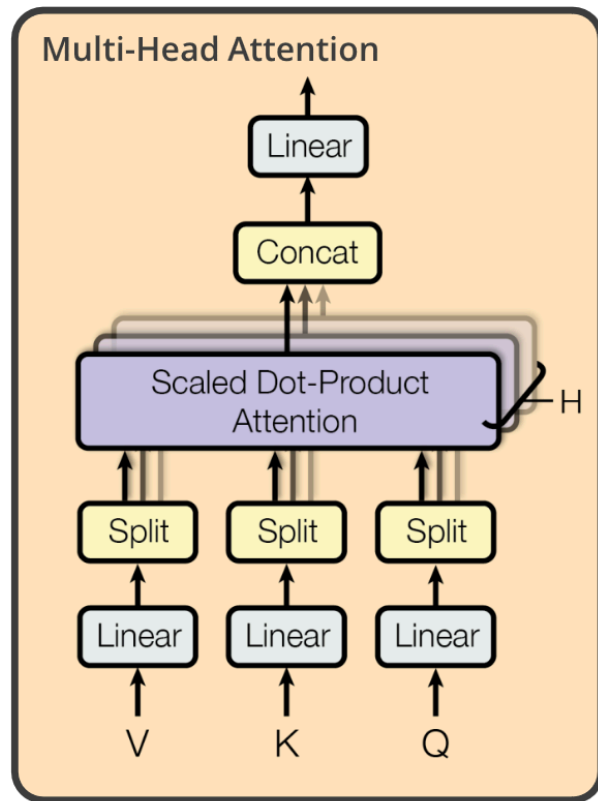


# Multi-Head Attention

$$q_i = Qx_i \text{ (queries)}$$

$$k_i = Kx_i \text{ (keys)}$$

$$v_i = Vx_i \text{ (values)}$$



$$\text{head}_1 = \text{Attention}(QW_1^Q, KW_1^K, VW_1^V)$$

⋮

$$\text{head}_H = \text{Attention}(QW_H^Q, KW_H^K, VW_H^V)$$

$$\text{MultiHeadAtt}(Q, K, V) = \text{Concat}(\text{head}_1, \dots, \text{head}_H)$$

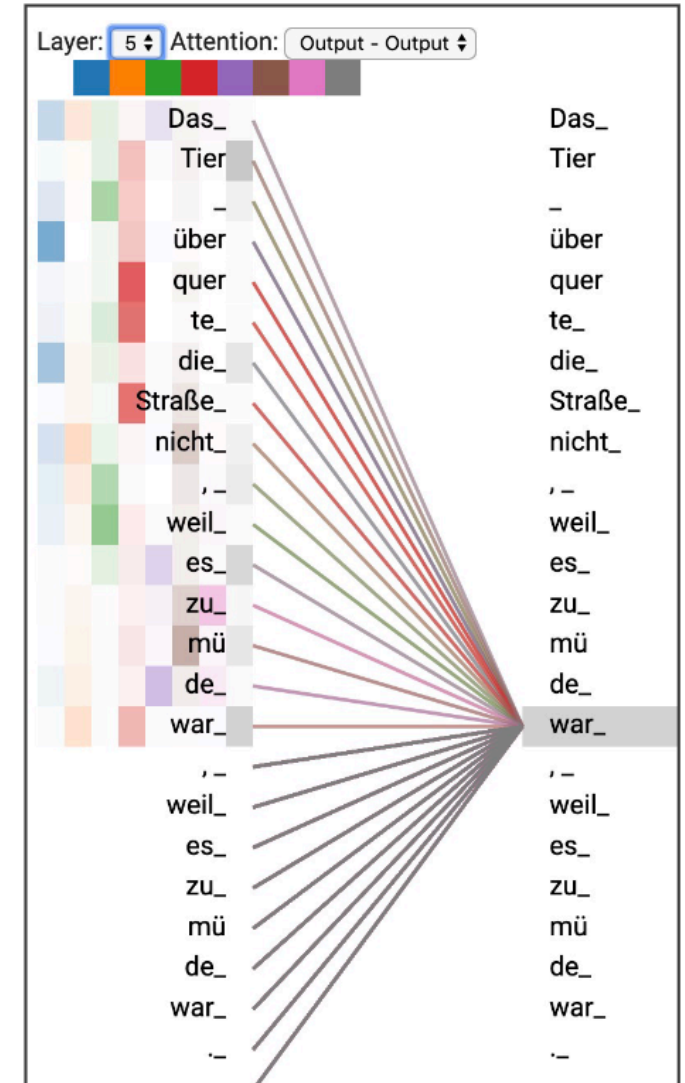
Inputs and outputs of each layer are the same dimensions:

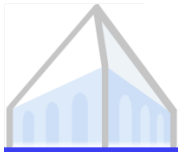
$$Q \in \mathbb{R}^{T \times d_{\text{model}}}$$

$$K \in \mathbb{R}^{T \times d_{\text{model}}}$$

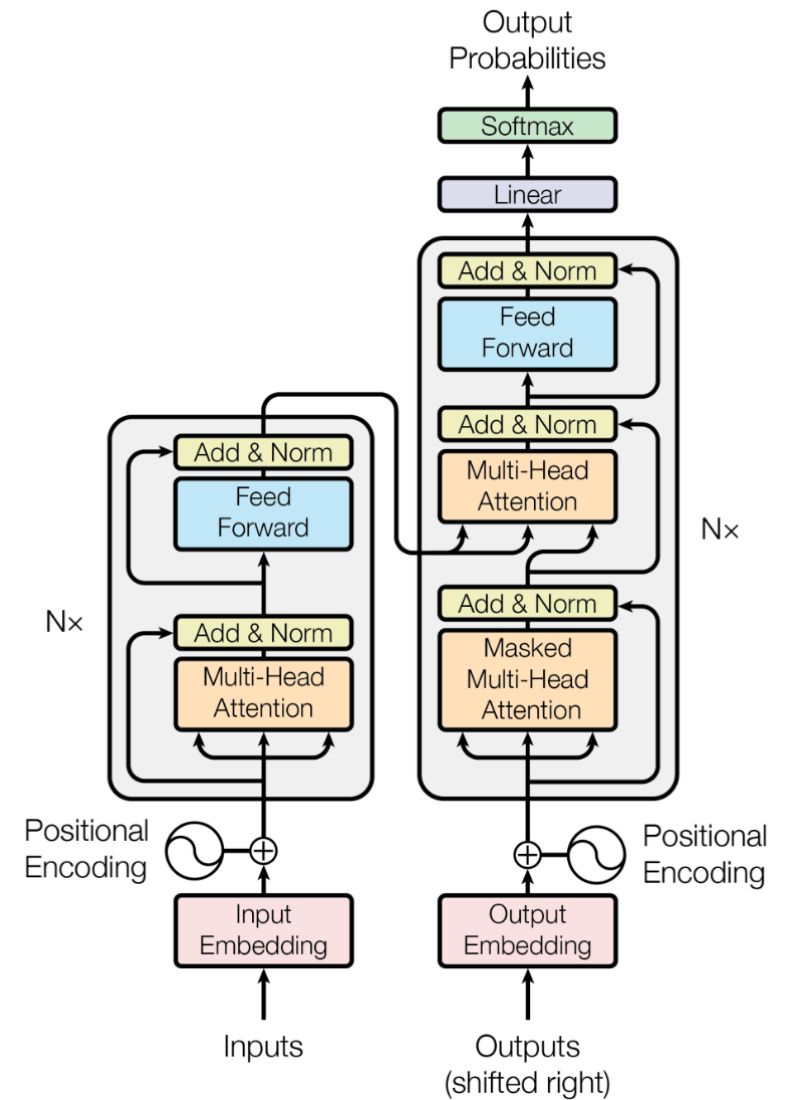
$$V \in \mathbb{R}^{T \times d_{\text{model}}}$$

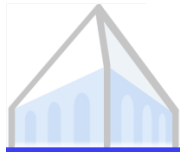
$$\text{MultiHeadAtt}(Q, K, V) \in \mathbb{R}^{T \times d_{\text{model}}}$$



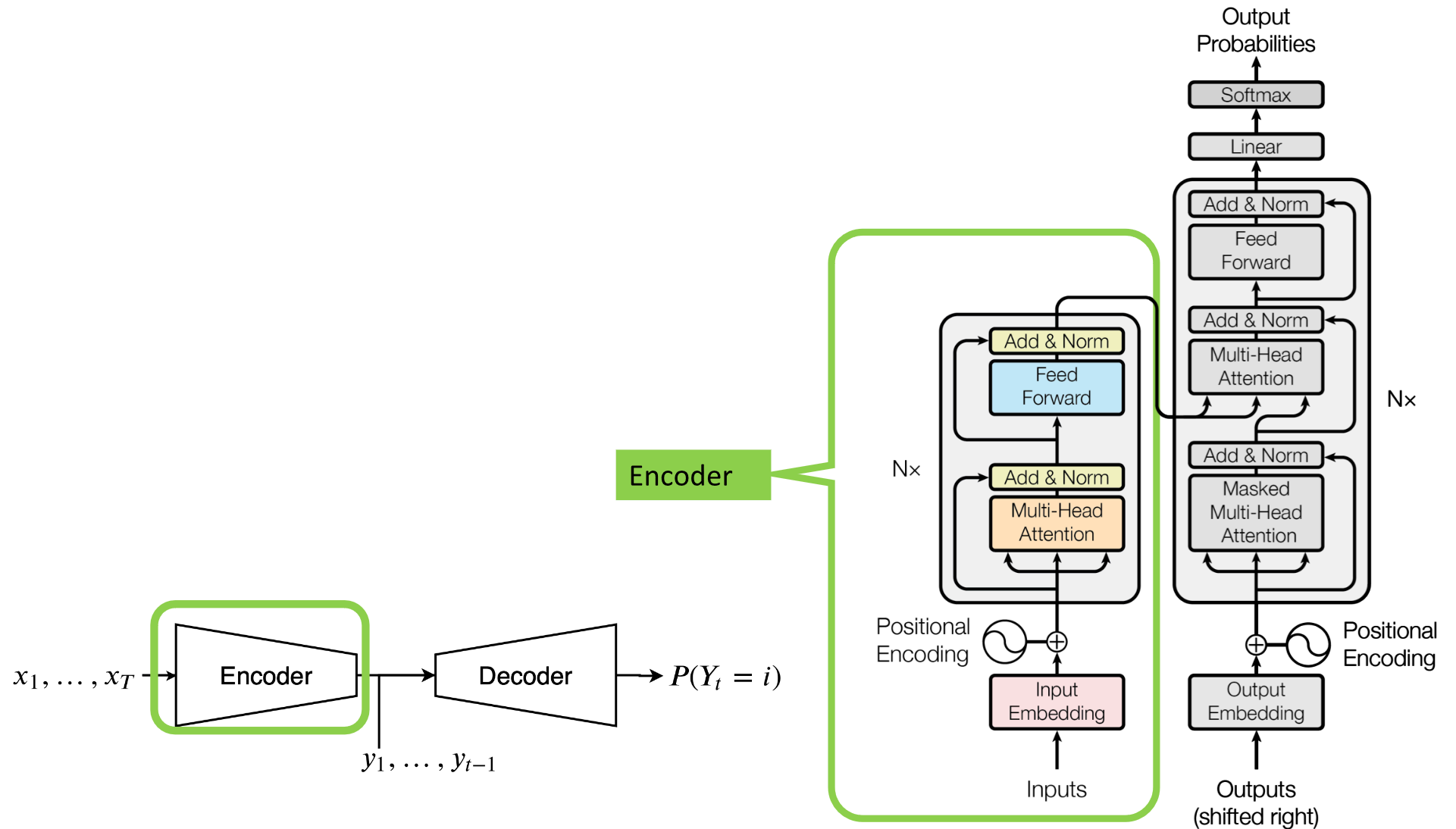


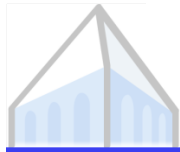
# Transformer



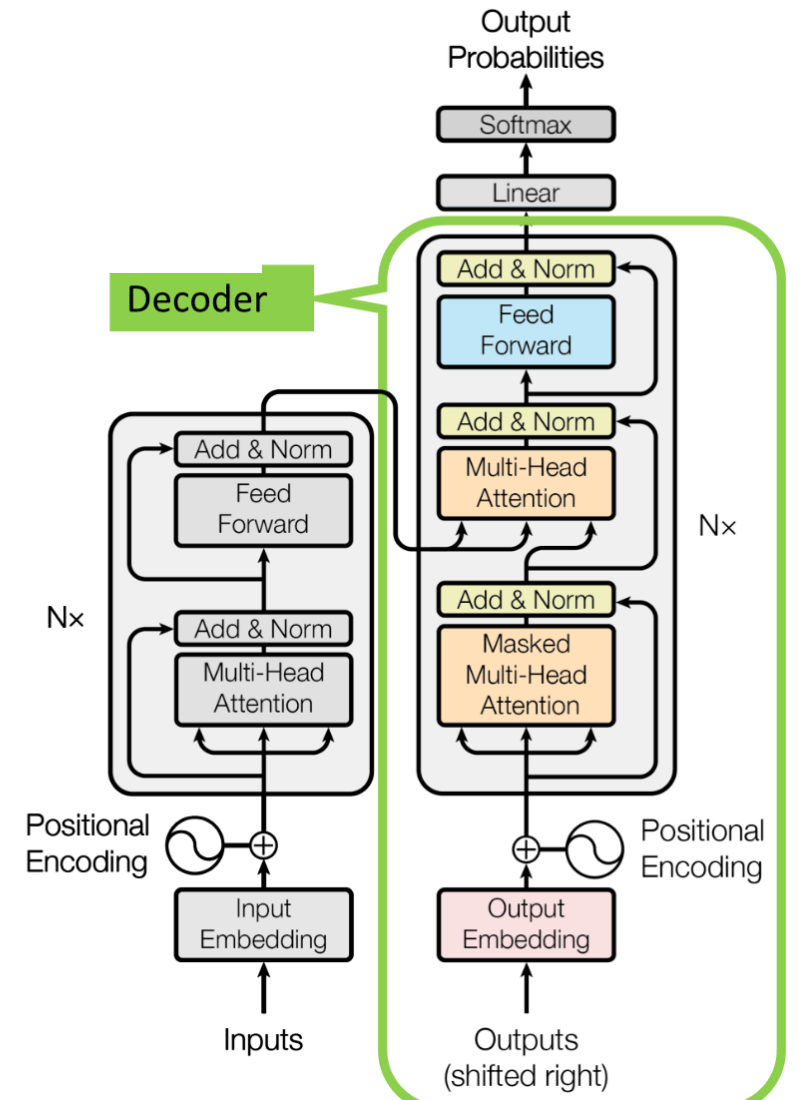
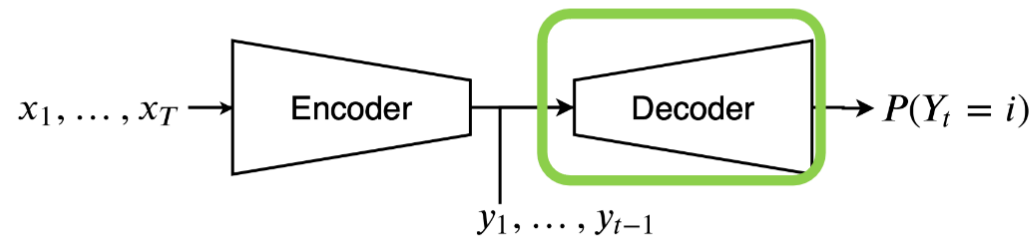


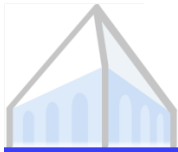
# Encoder





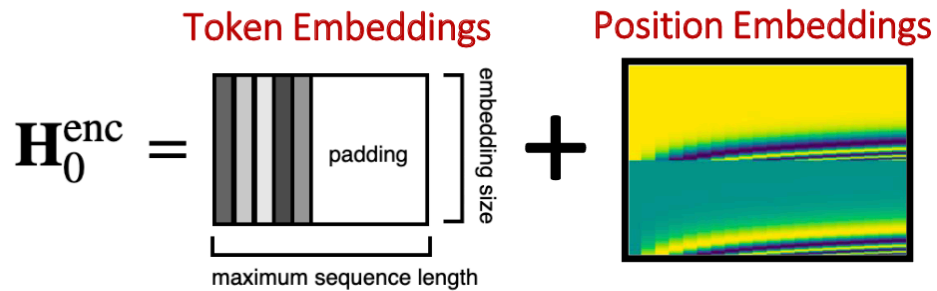
# Decoder



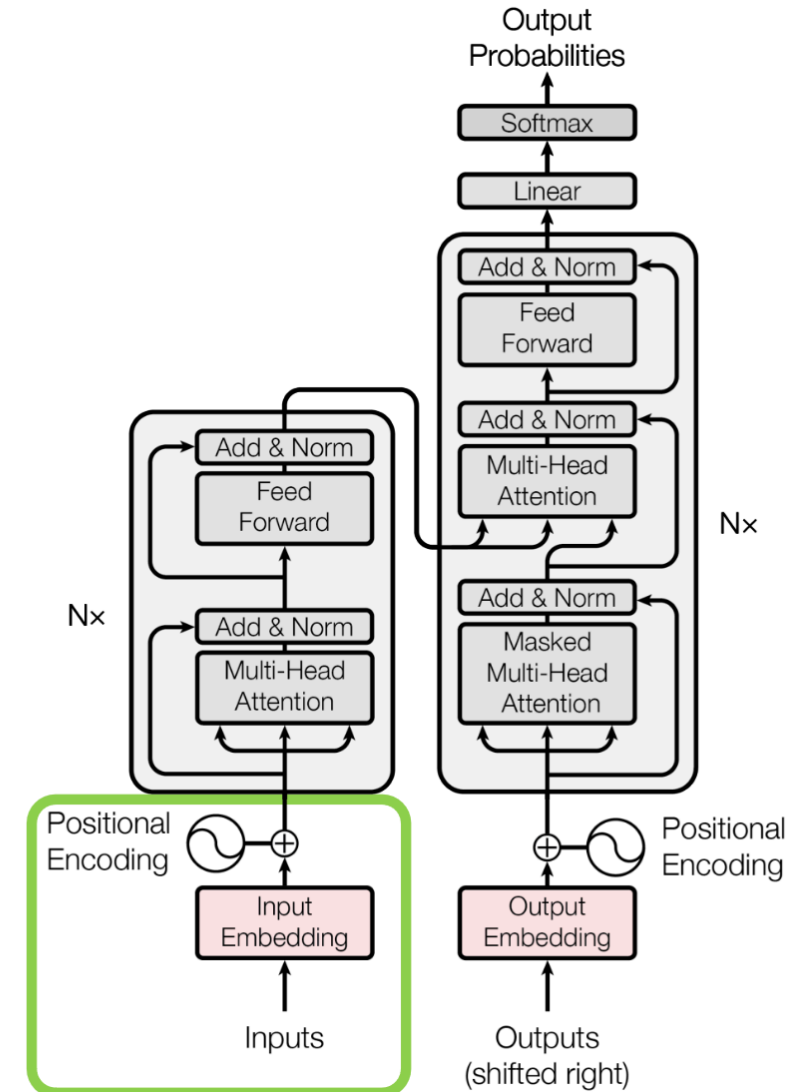
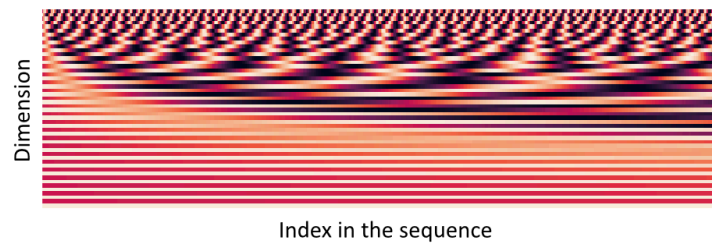


# Encoder Input

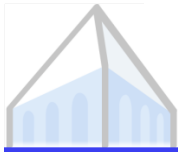
The input into the encoder looks like:



$$\mathbf{p}_i = \begin{pmatrix} \sin(i/10000^{2*1/d}) \\ \cos(i/10000^{2*1/d}) \\ \vdots \\ \sin(i/10000^{2*d/2/d}) \\ \cos(i/10000^{2*d/2/d}) \end{pmatrix}$$

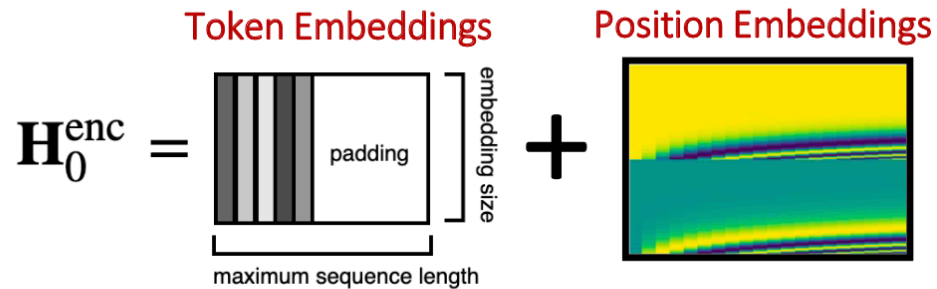




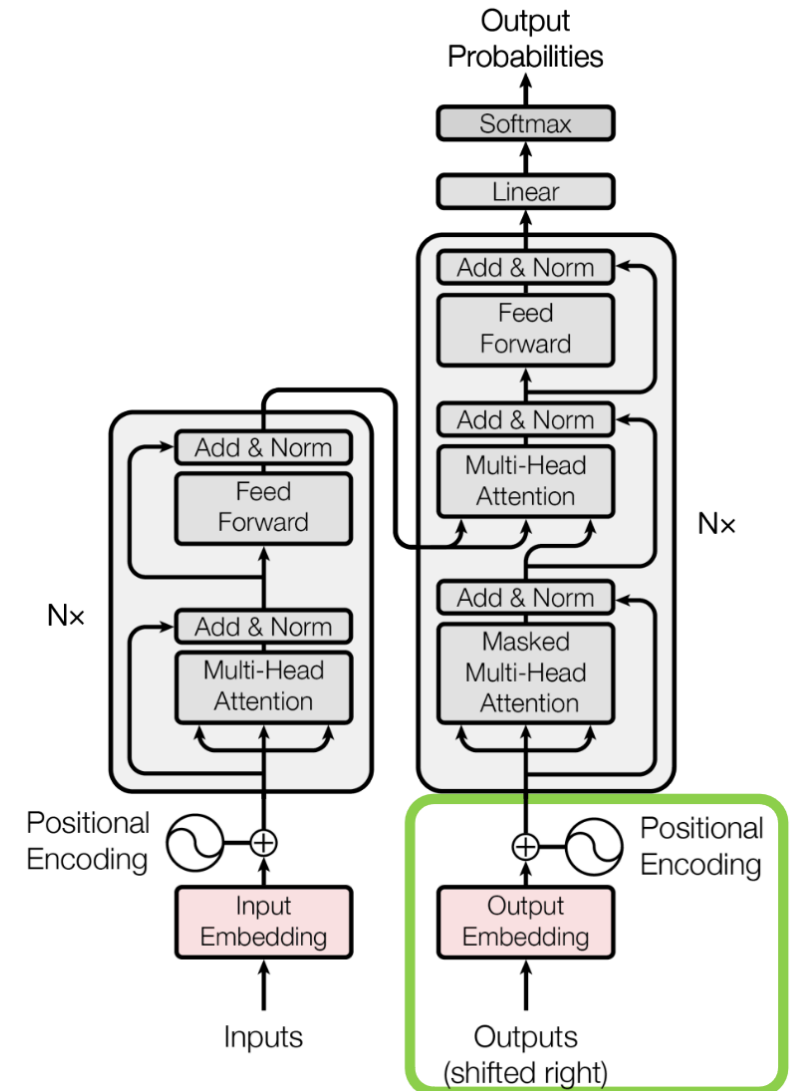
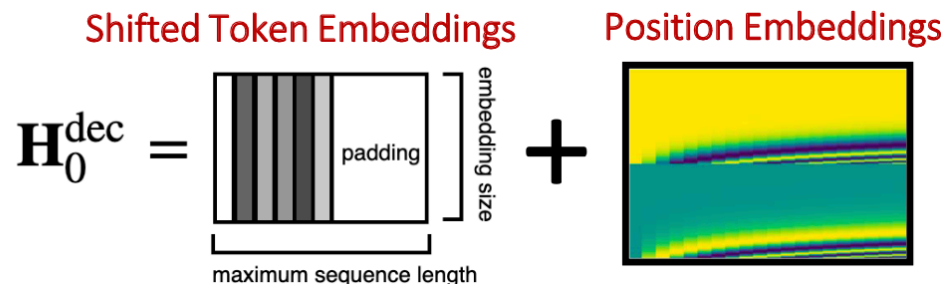


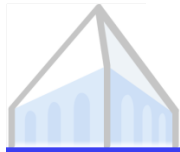
# Decoder Input

The input into the encoder looks like:

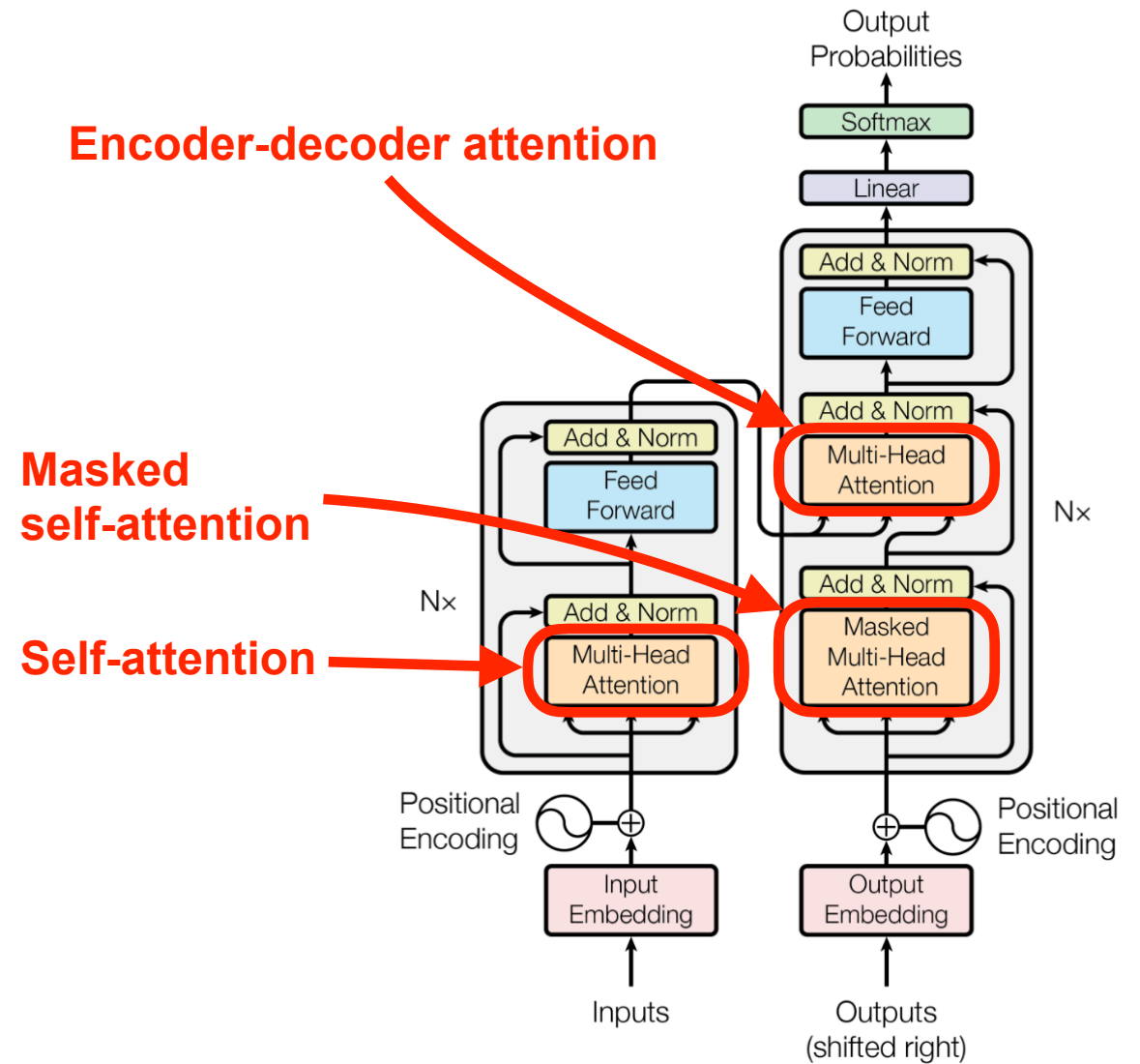


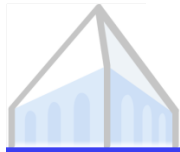
The input to the decoder looks like:





# Attention

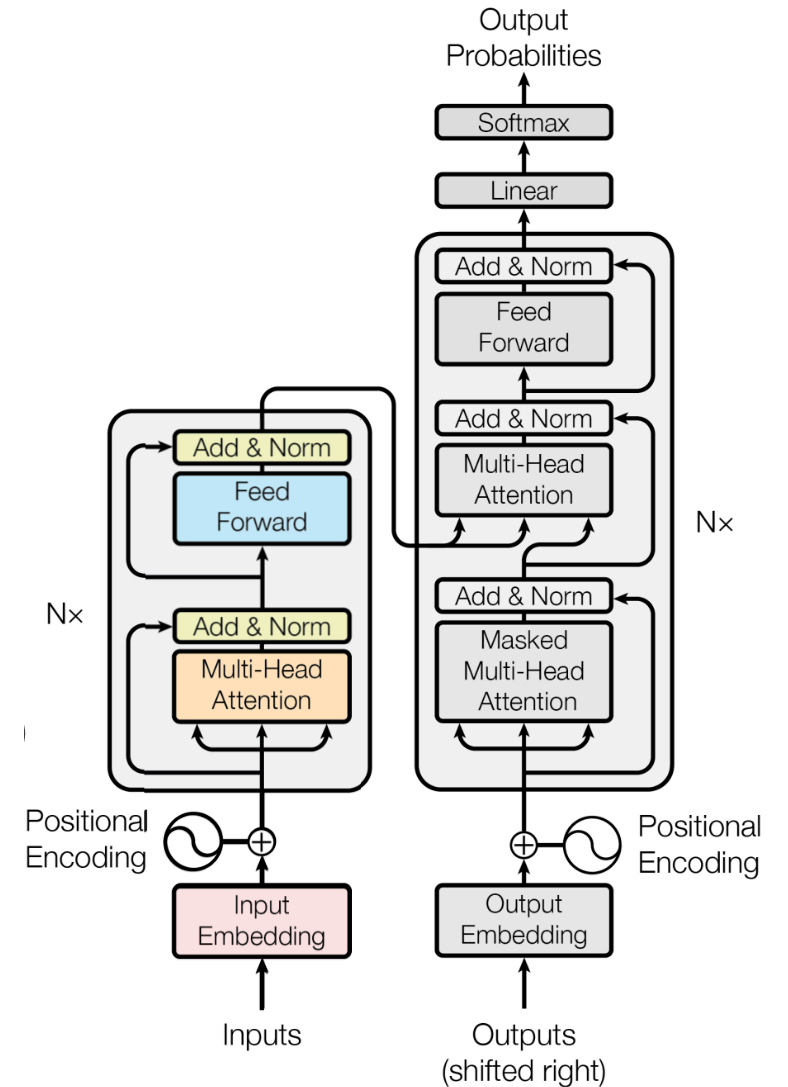


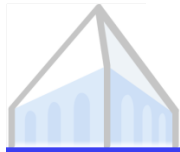


# Encoder

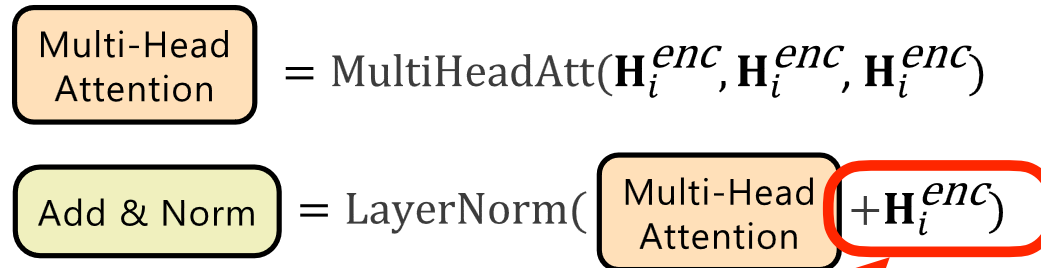
Multi-Head Attention

$$= \text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \mathbf{H}_i^{enc})$$

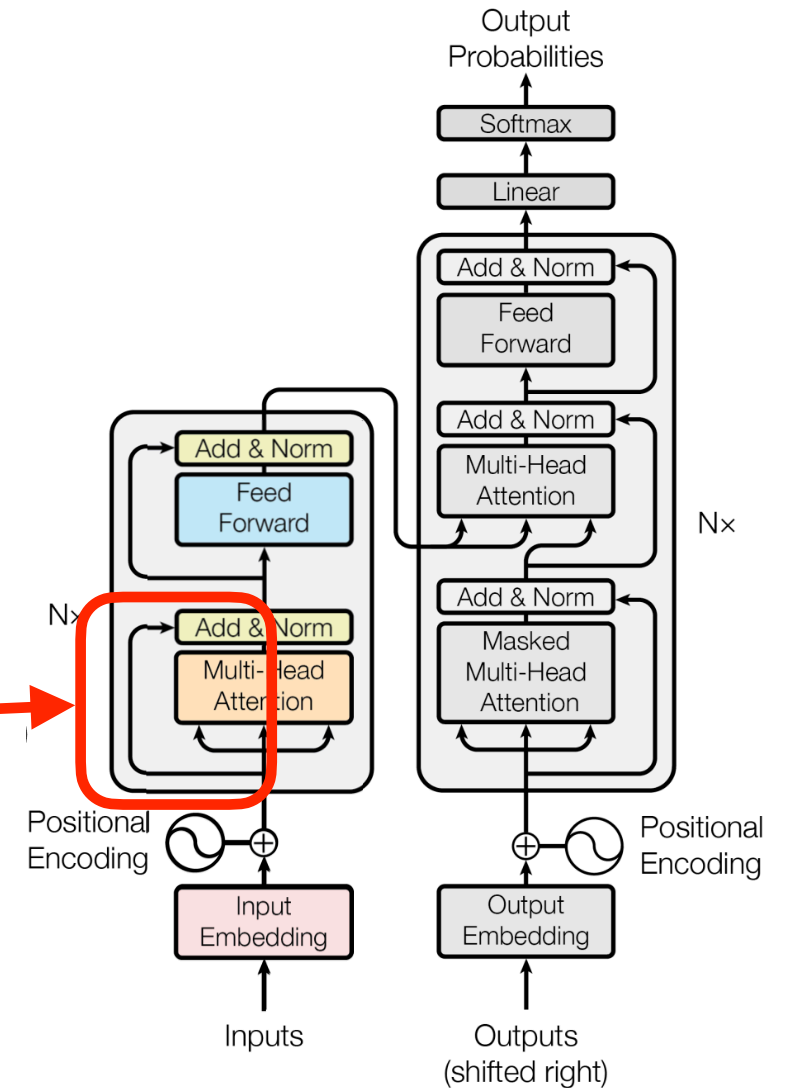


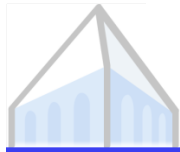


# Encoder

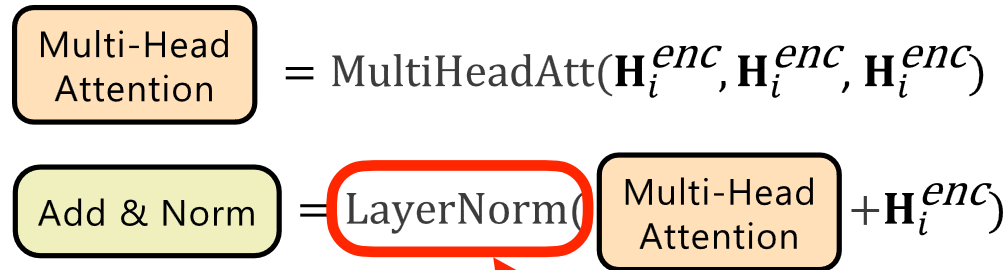


**Residual connection**





# Encoder



Let  $x \in \mathbb{R}^d$  be an individual (word) vector in the model.

Let  $\mu = \sum_{j=1}^d x_j$ ; this is the mean;  $\mu \in \mathbb{R}$ .

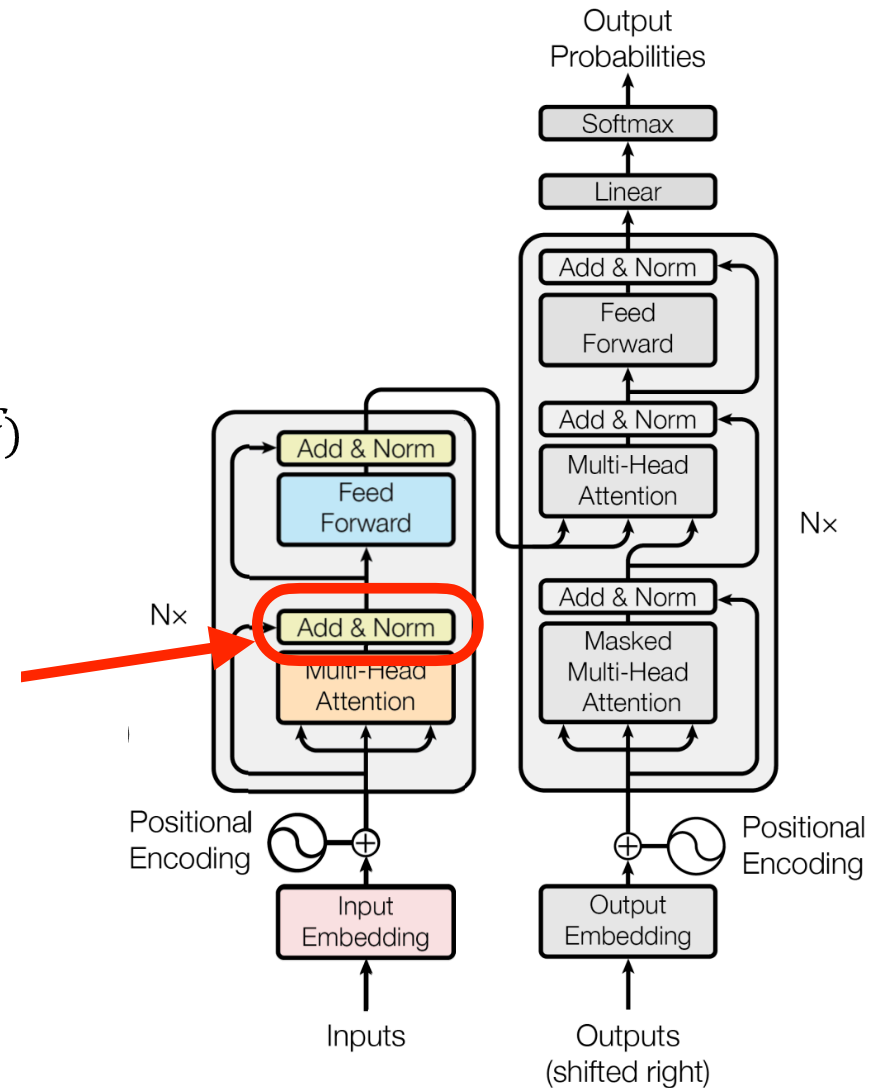
Let  $\sigma = \sqrt{\frac{1}{d} \sum_{j=1}^d (x_j - \mu)^2}$ ; this is the standard deviation;  $\sigma \in \mathbb{R}$ .

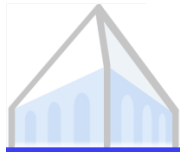
Let  $\gamma \in \mathbb{R}^d$  and  $\beta \in \mathbb{R}^d$  be learned “gain” and “bias” parameters. (Can omit!)

Then layer normalization computes:

$$\text{output} = \frac{x - \mu}{\sqrt{\sigma + \epsilon}} * \gamma + \beta$$

Normalize by scalar mean and variance  $\rightarrow$   $\frac{x - \mu}{\sqrt{\sigma + \epsilon}}$   $\leftarrow$  Modulate by learned elementwise gain and bias  $* \gamma + \beta$



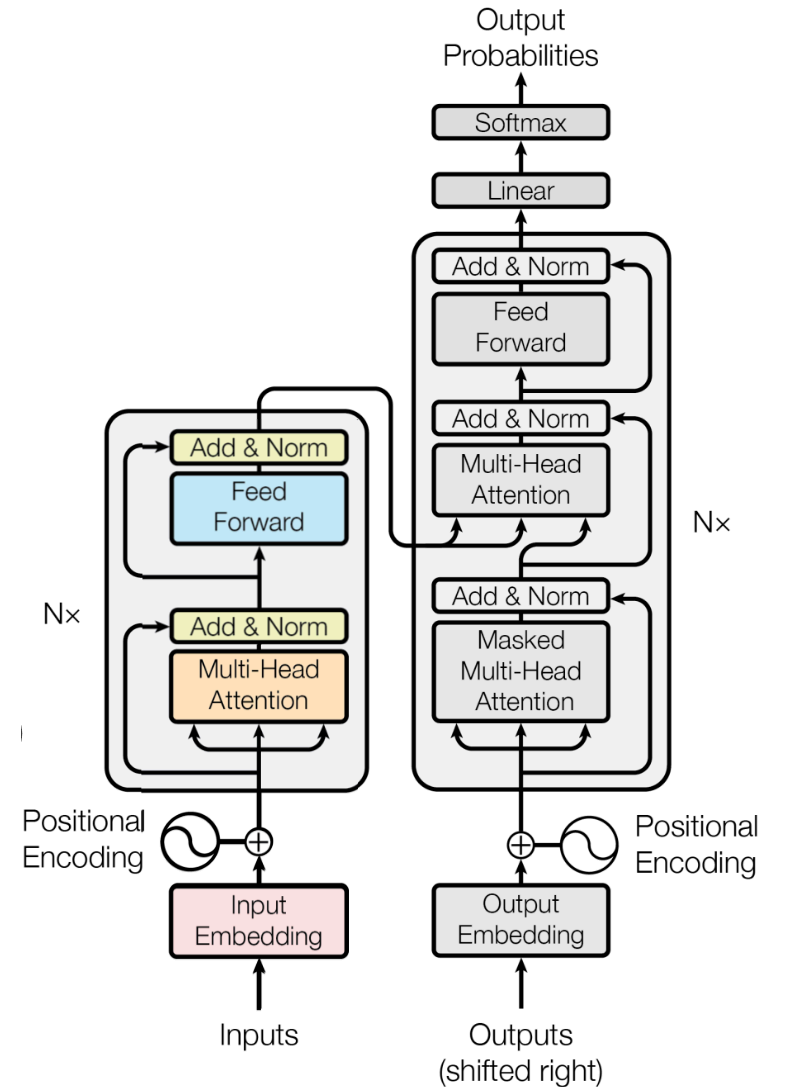


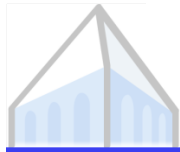
# Encoder

Multi-Head Attention =  $\text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \mathbf{H}_i^{enc})$

Add & Norm =  $\text{LayerNorm}(\text{Multi-Head Attention} + \mathbf{H}_i^{enc})$

Feed Forward =  $\max(0, \text{Add & Norm} \mathbf{W}_1 + b_1) \mathbf{W}_2 + b_2$





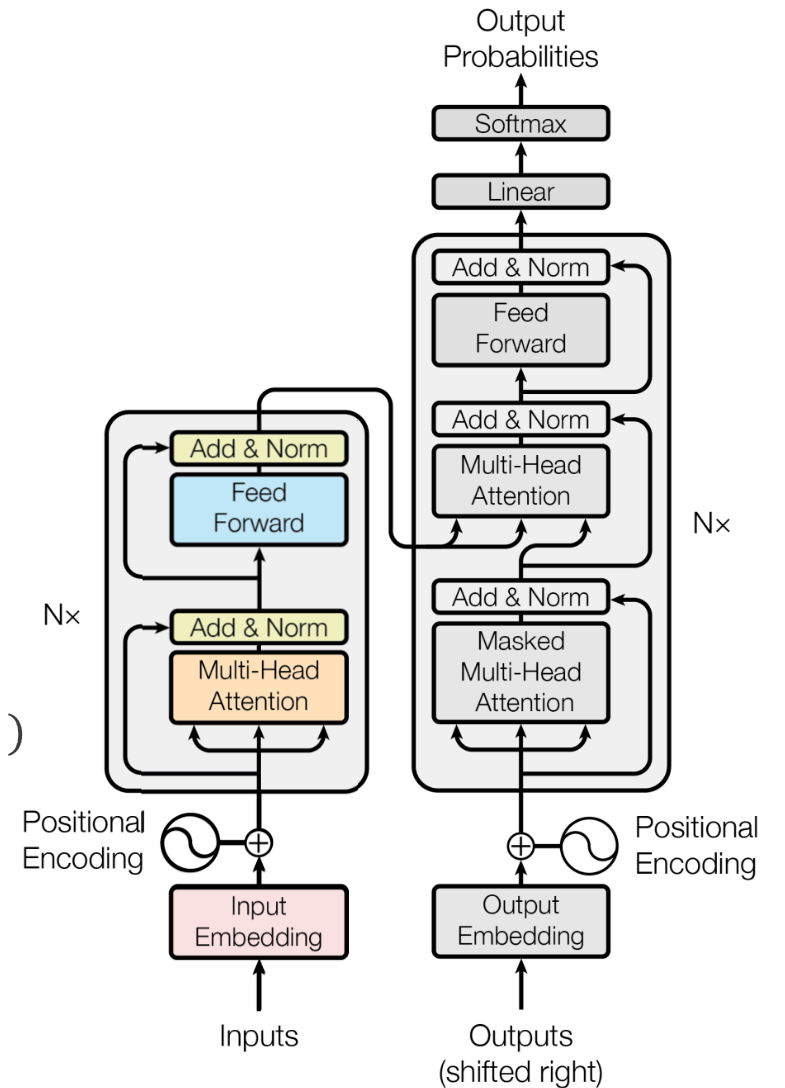
# Encoder

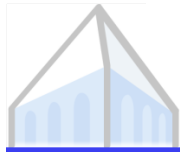
Multi-Head Attention =  $\text{MultiHeadAtt}(\mathbf{H}_i^{enc}, \mathbf{H}_i^{enc}, \mathbf{H}_i^{enc})$

Add & Norm =  $\text{LayerNorm}(\text{Multi-Head Attention} + \mathbf{H}_i^{enc})$

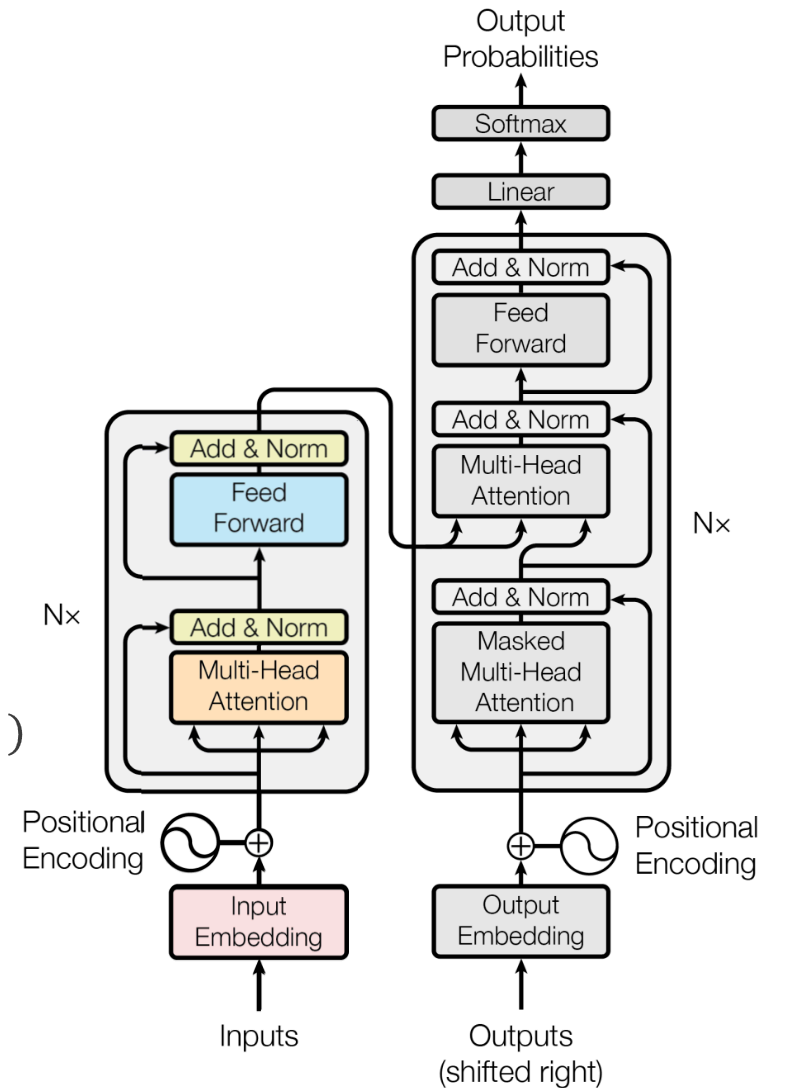
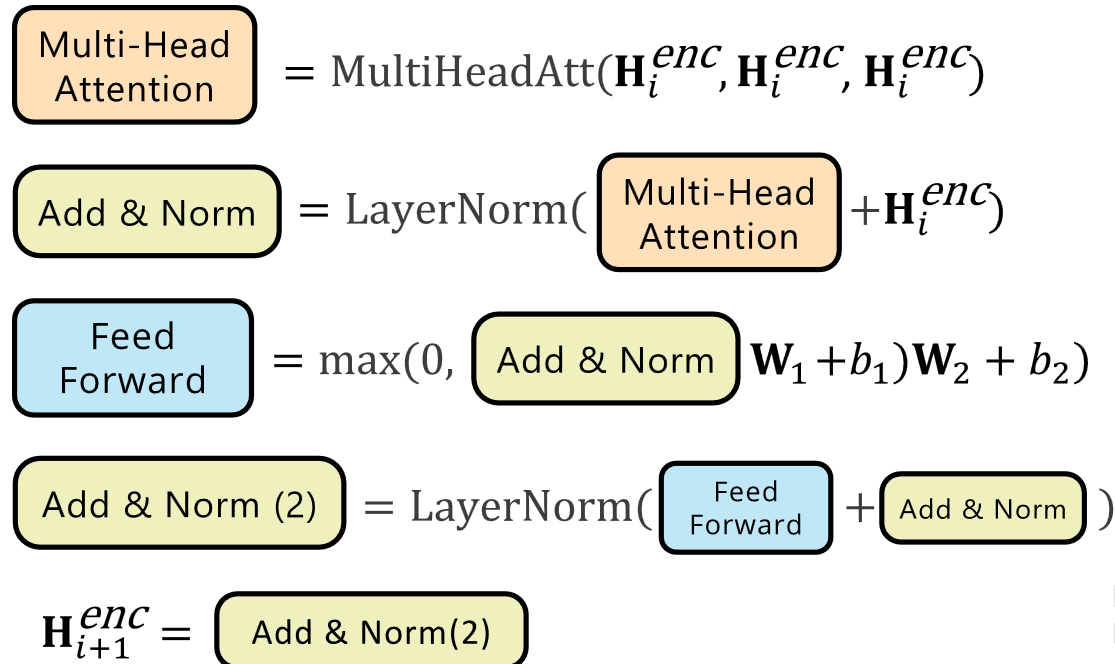
Feed Forward =  $\max(0, \text{Add \& Norm } \mathbf{W}_1 + b_1) \mathbf{W}_2 + b_2$

Add & Norm (2) =  $\text{LayerNorm}(\text{Feed Forward} + \text{Add \& Norm})$

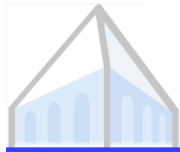




# Encoder

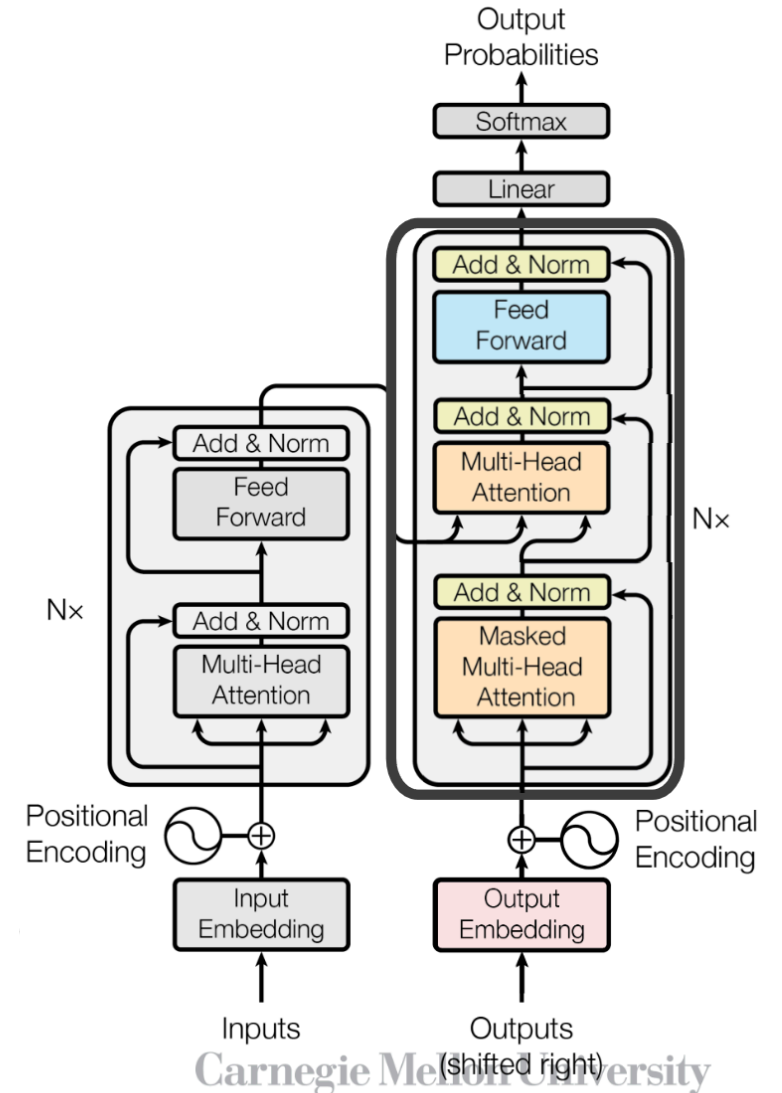


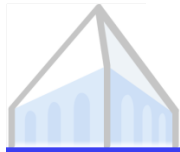




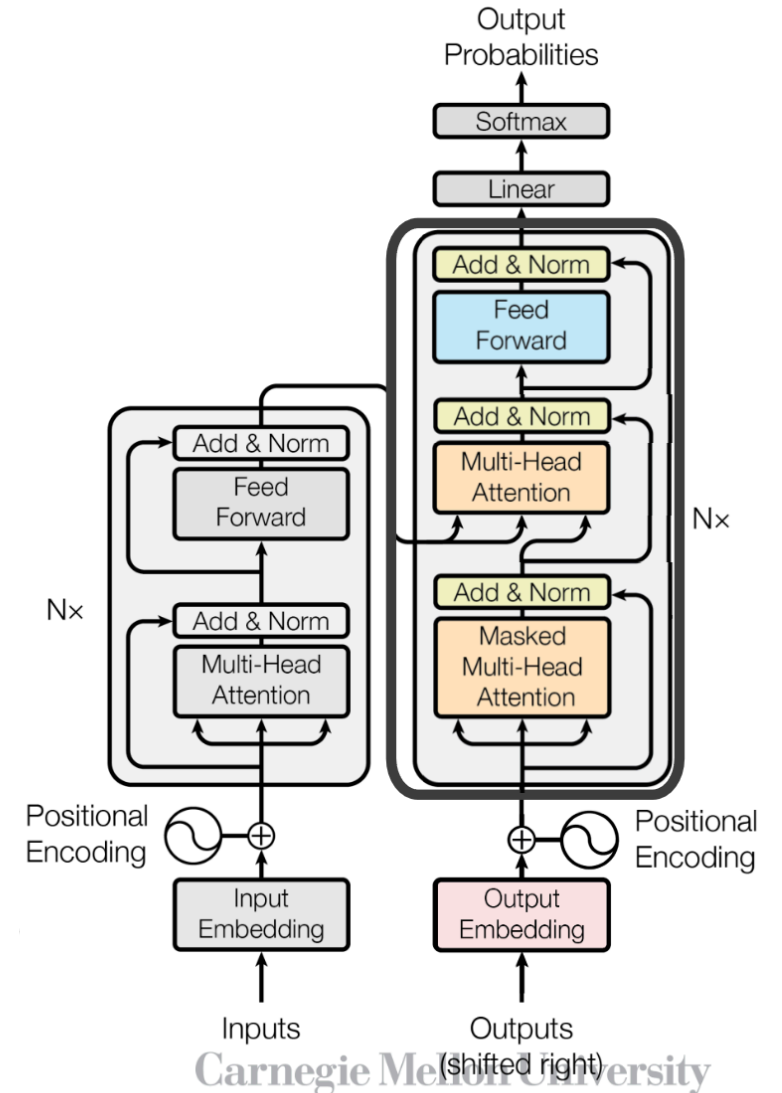
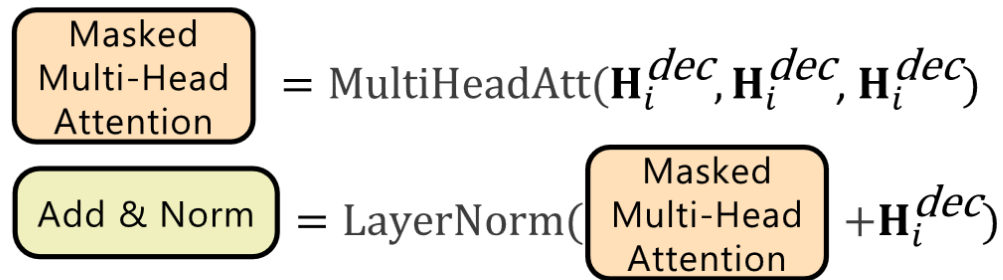
# Decoder

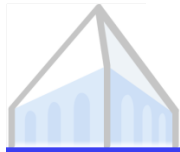
Masked Multi-Head Attention =  $\text{MultiHeadAtt}(\mathbf{H}_i^{dec}, \mathbf{H}_i^{dec}, \mathbf{H}_i^{dec})$



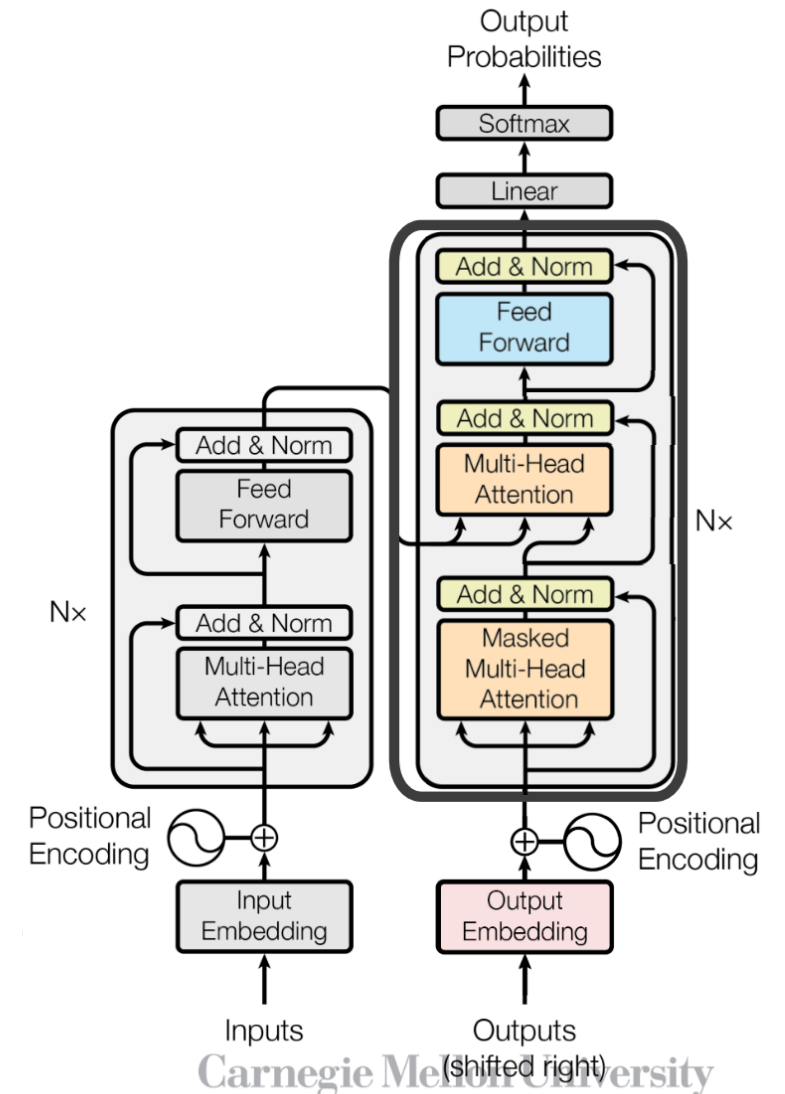
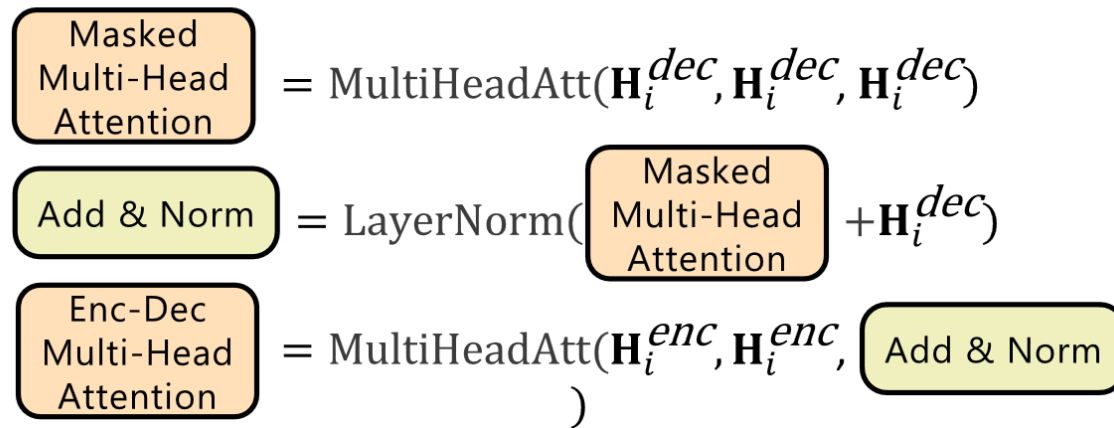


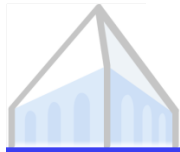
# Decoder



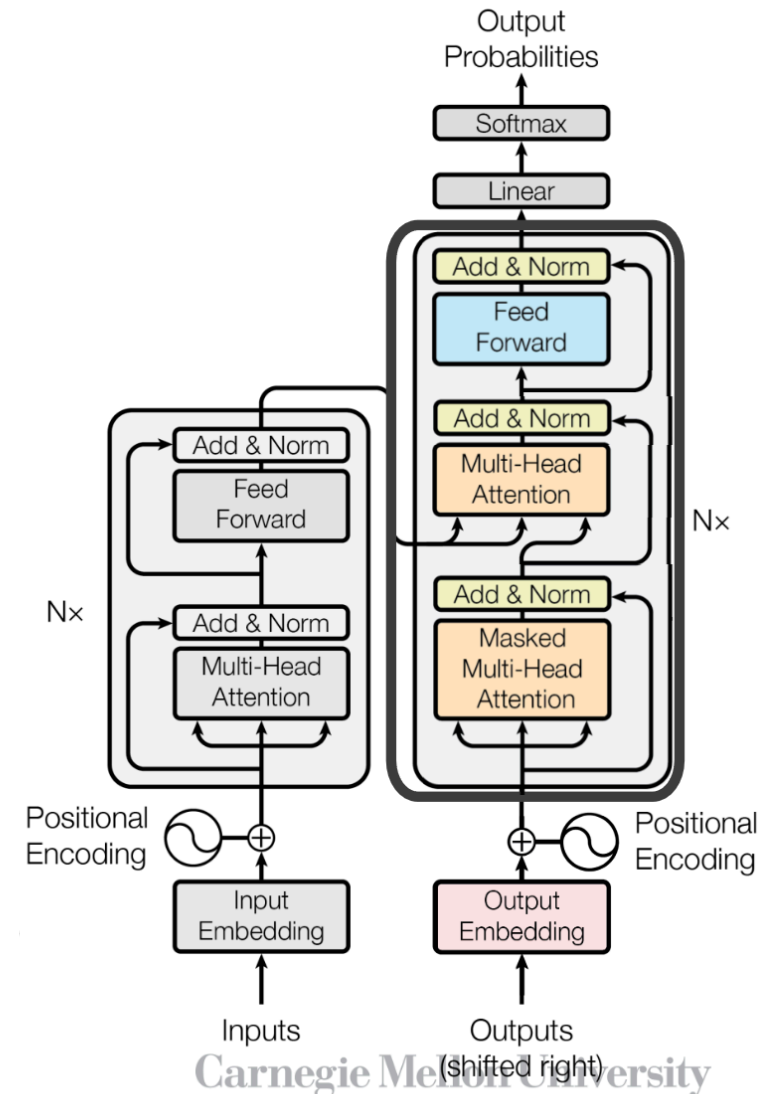
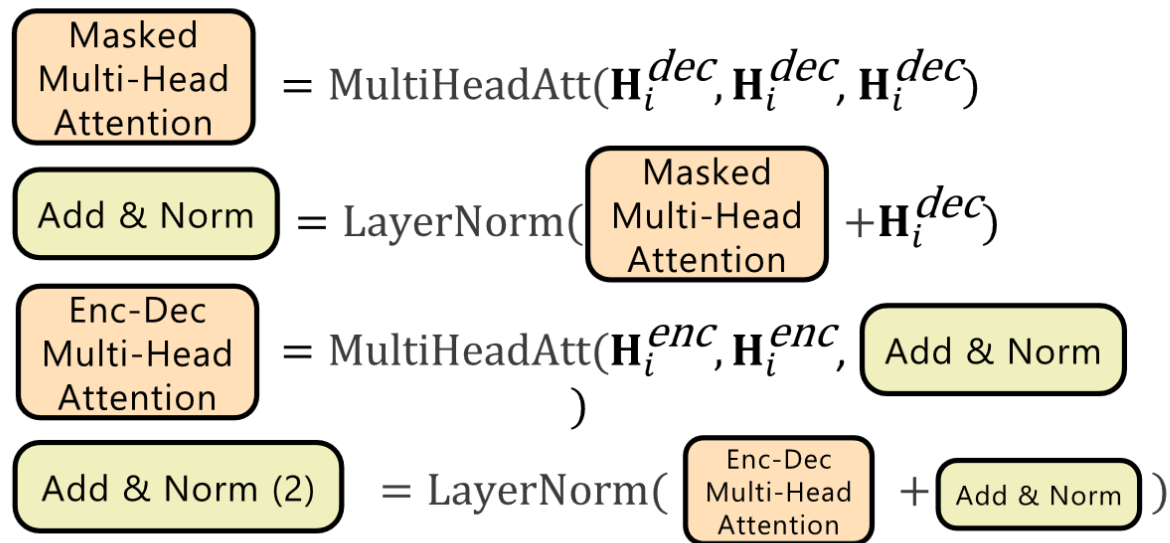


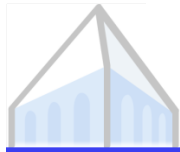
# Decoder



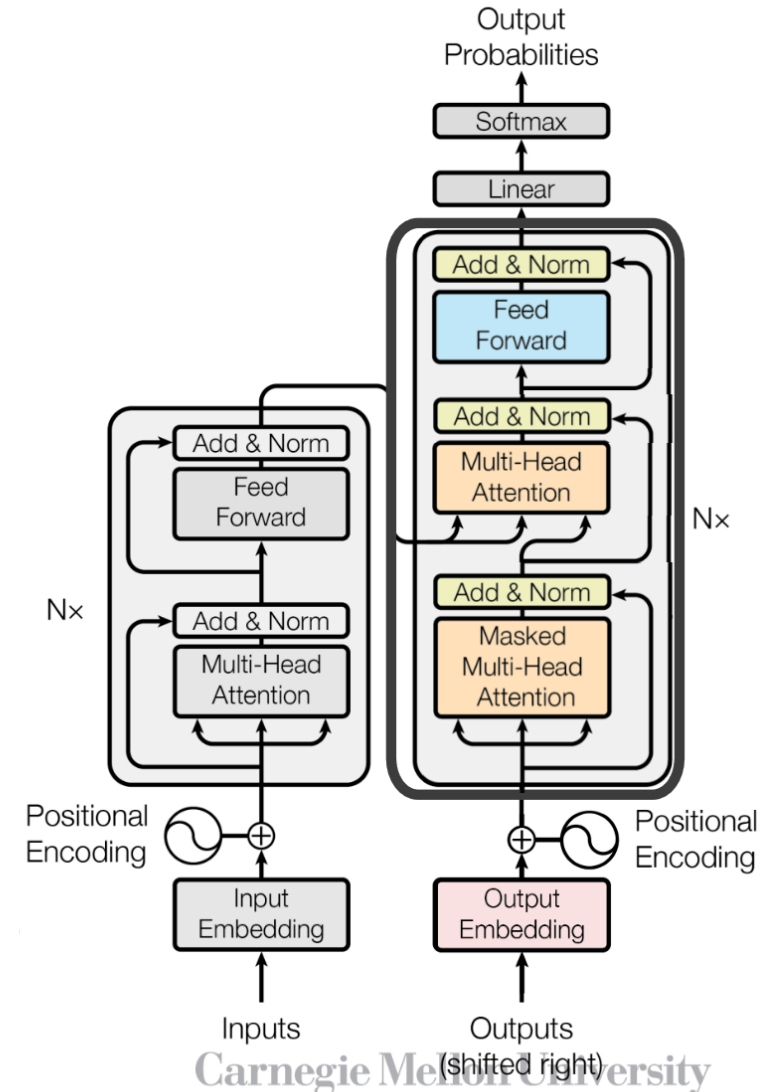
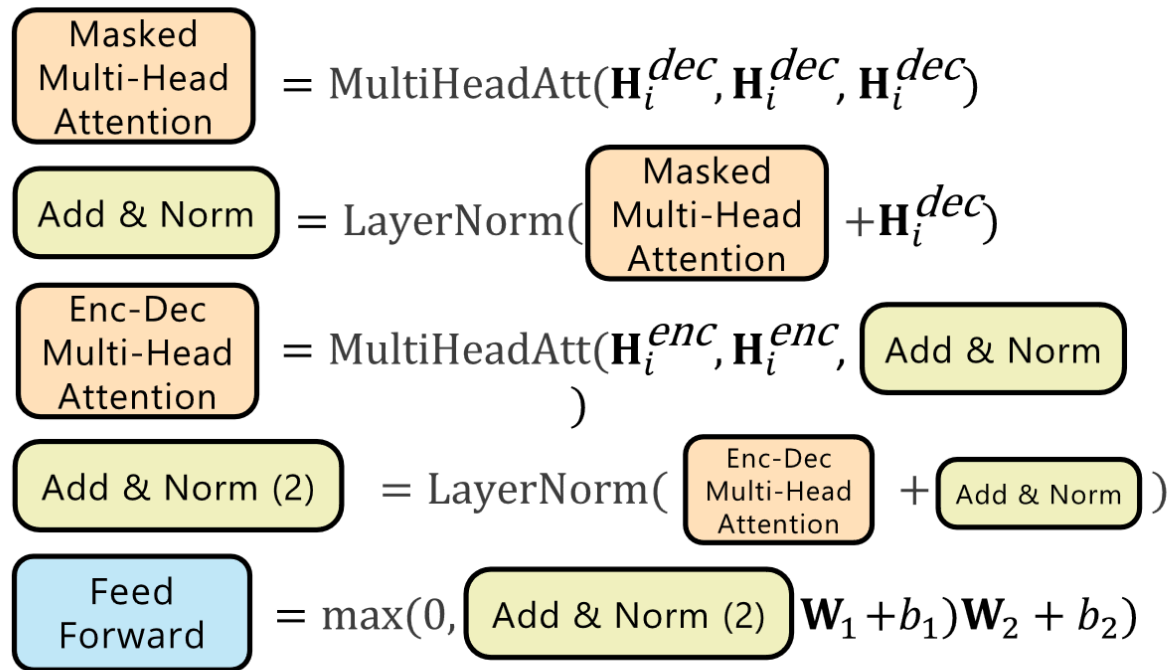


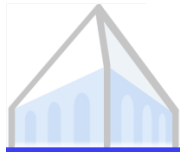
# Decoder



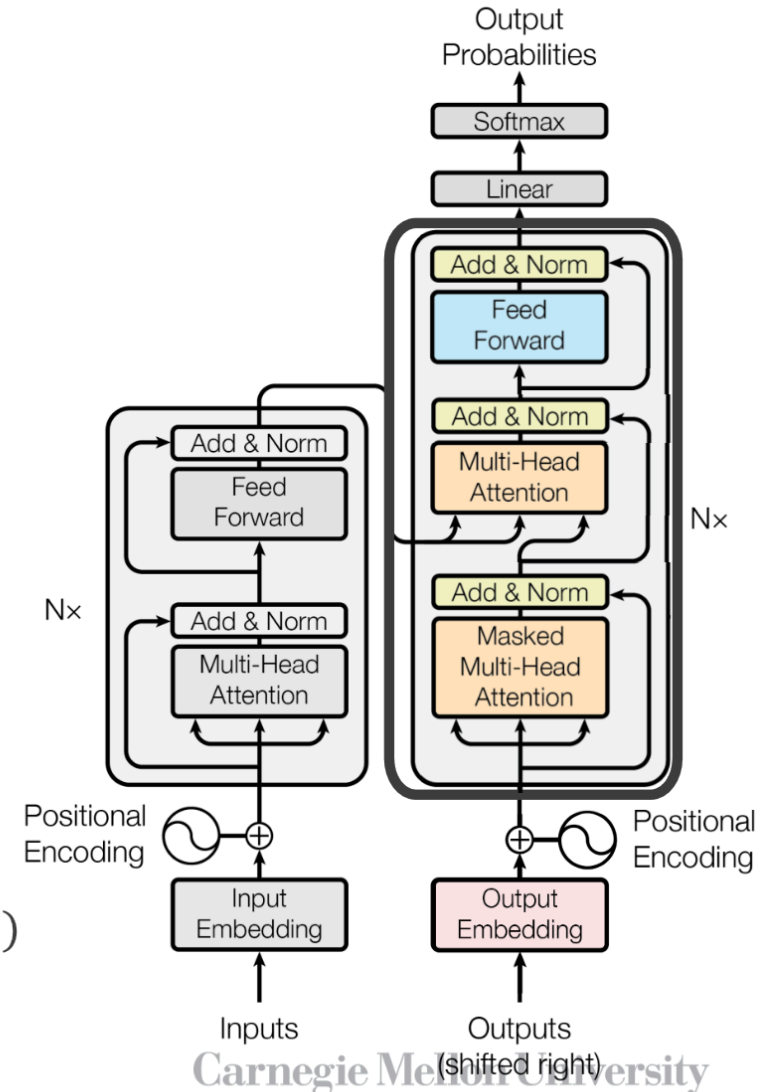
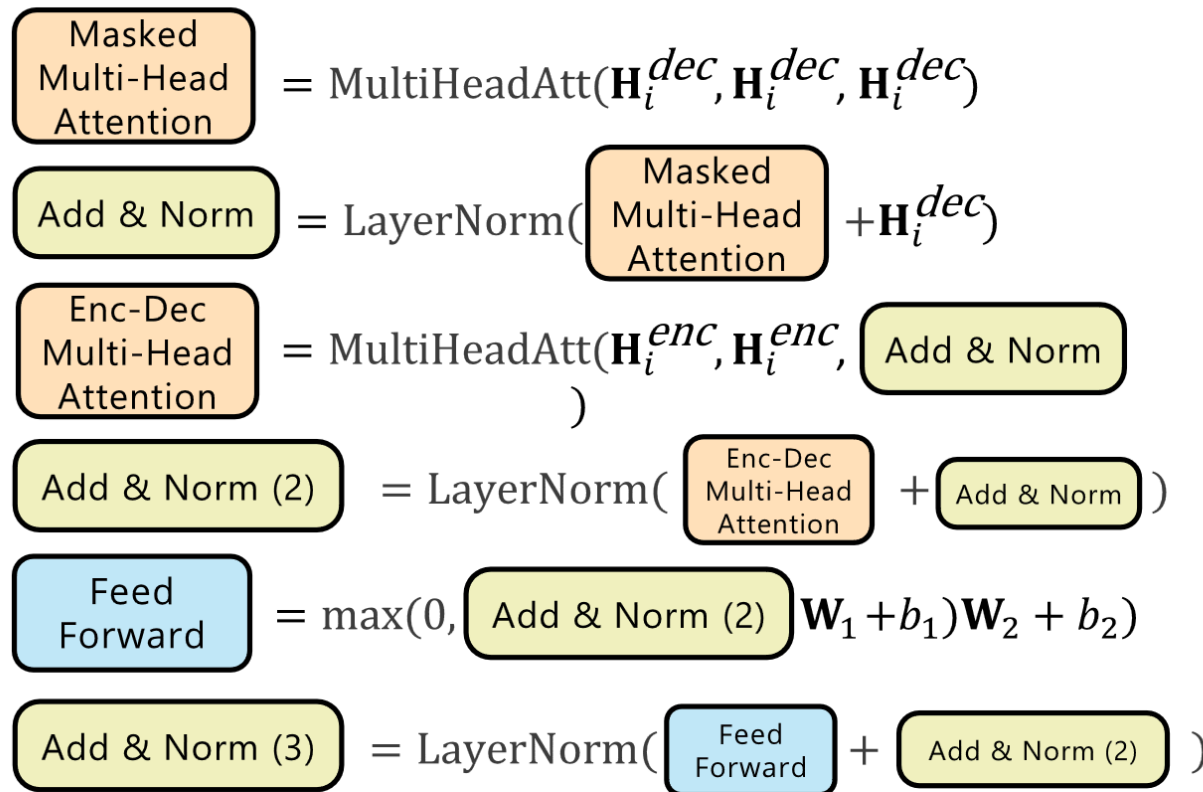


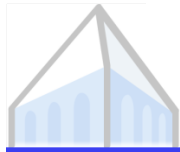
# Decoder



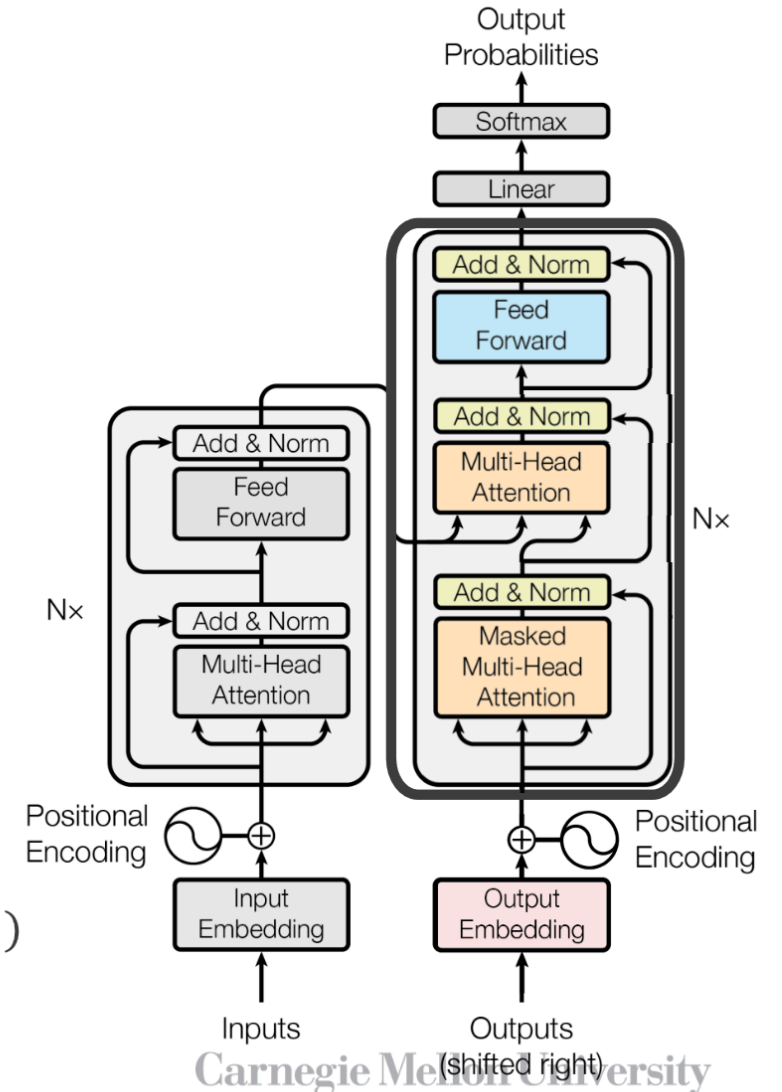
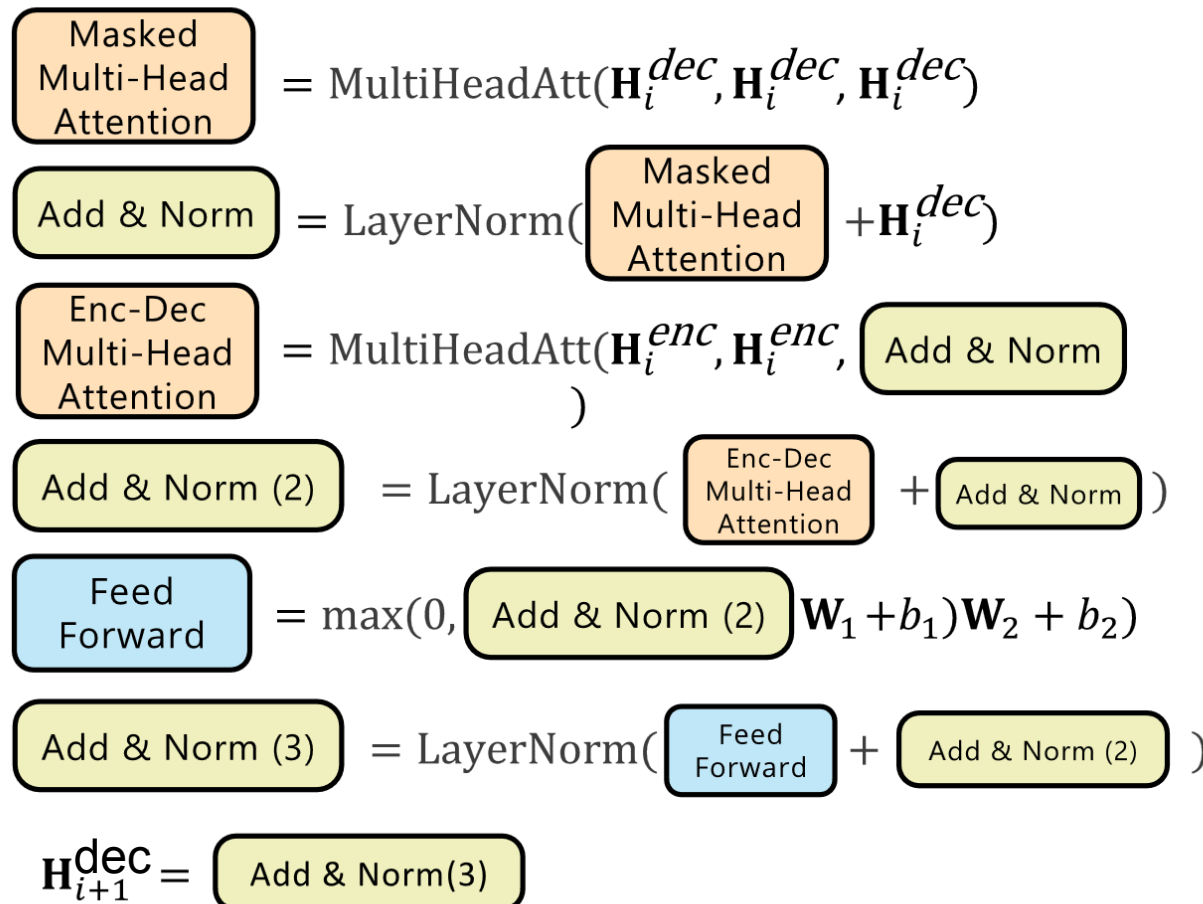


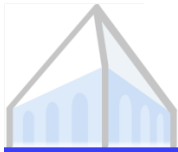
# Decoder



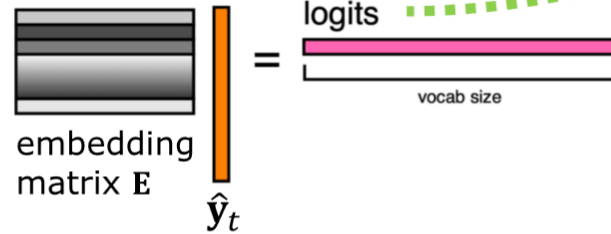


# Decoder

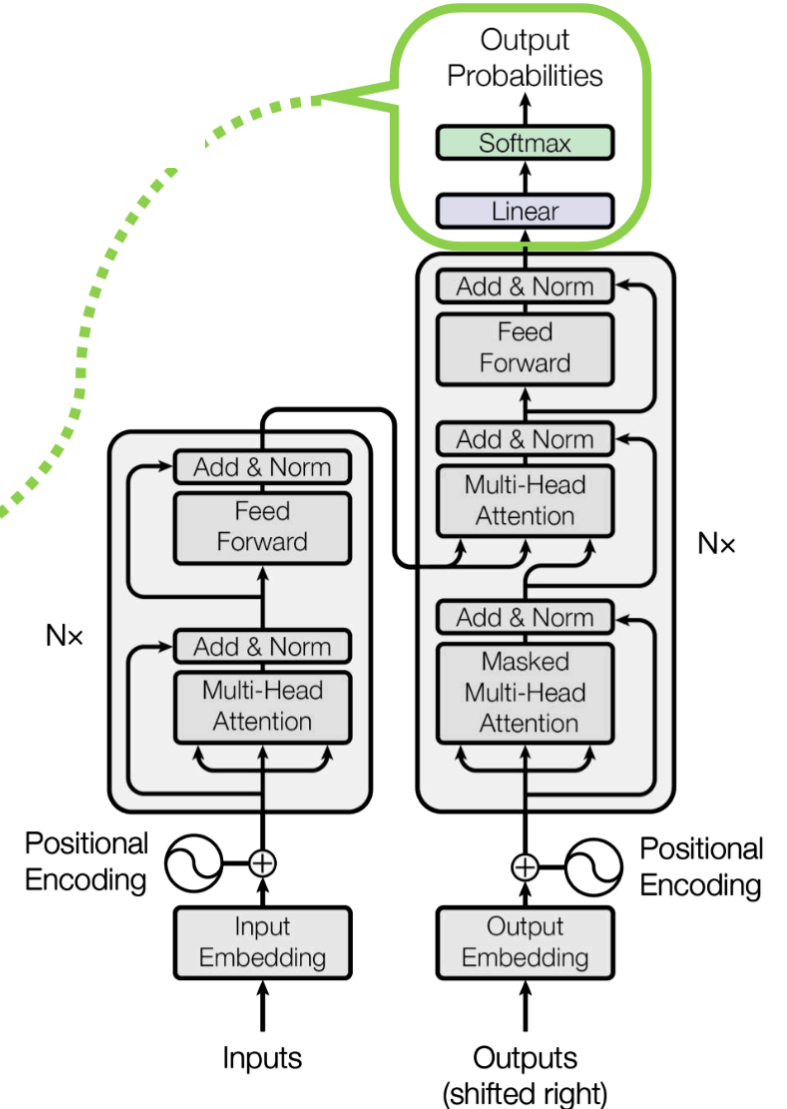




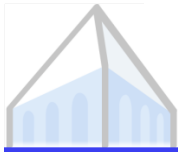
# Output Probabilities



$$P(Y_t = i | \mathbf{x}_{1:T}, \mathbf{y}_{1:t-1}) = \frac{\exp(\mathbf{E}\hat{\mathbf{y}}_t[i])}{\sum_j \exp(\mathbf{E}\hat{\mathbf{y}}_t[j])}$$

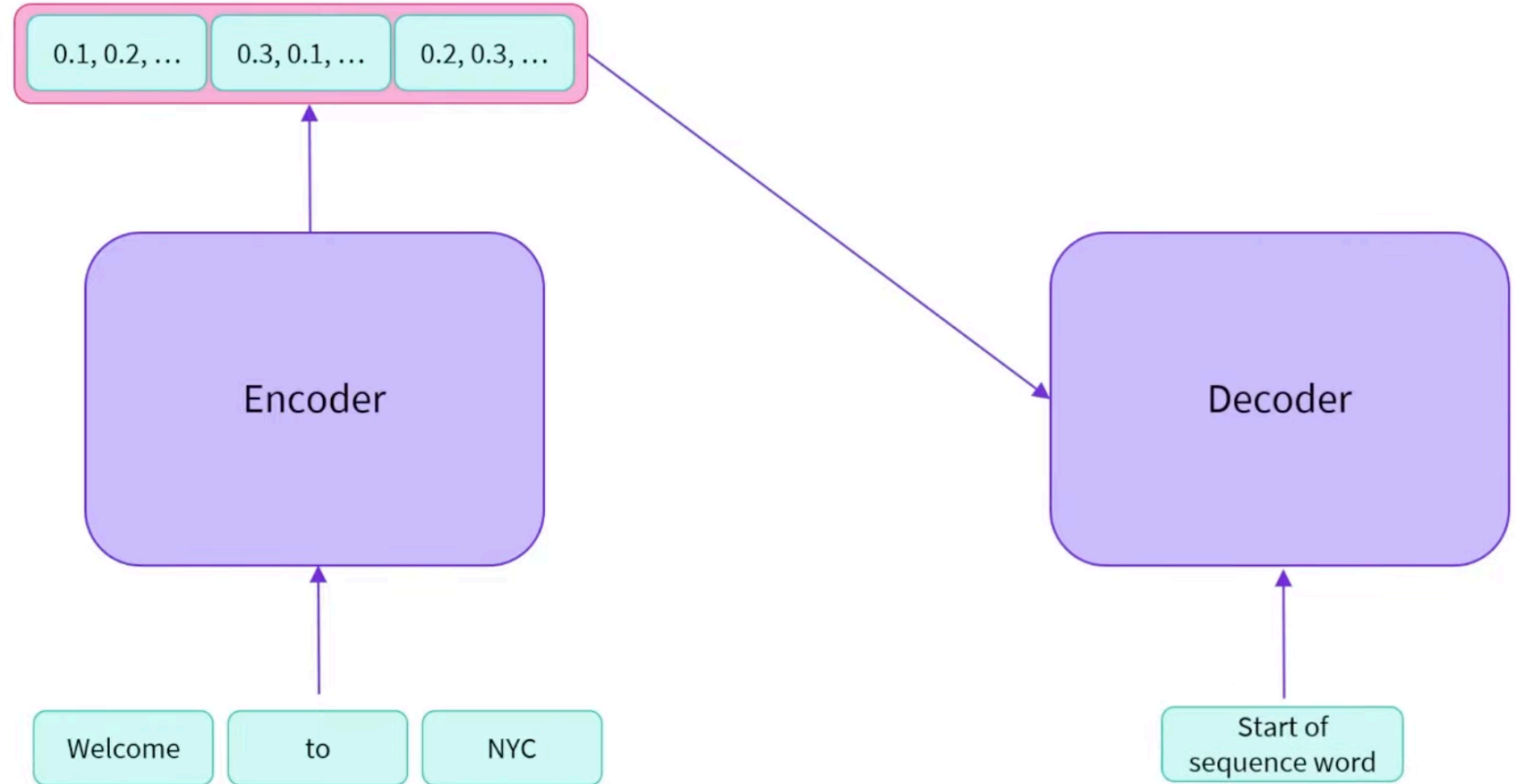


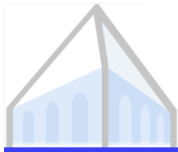




# Encoder-Decoder Inference

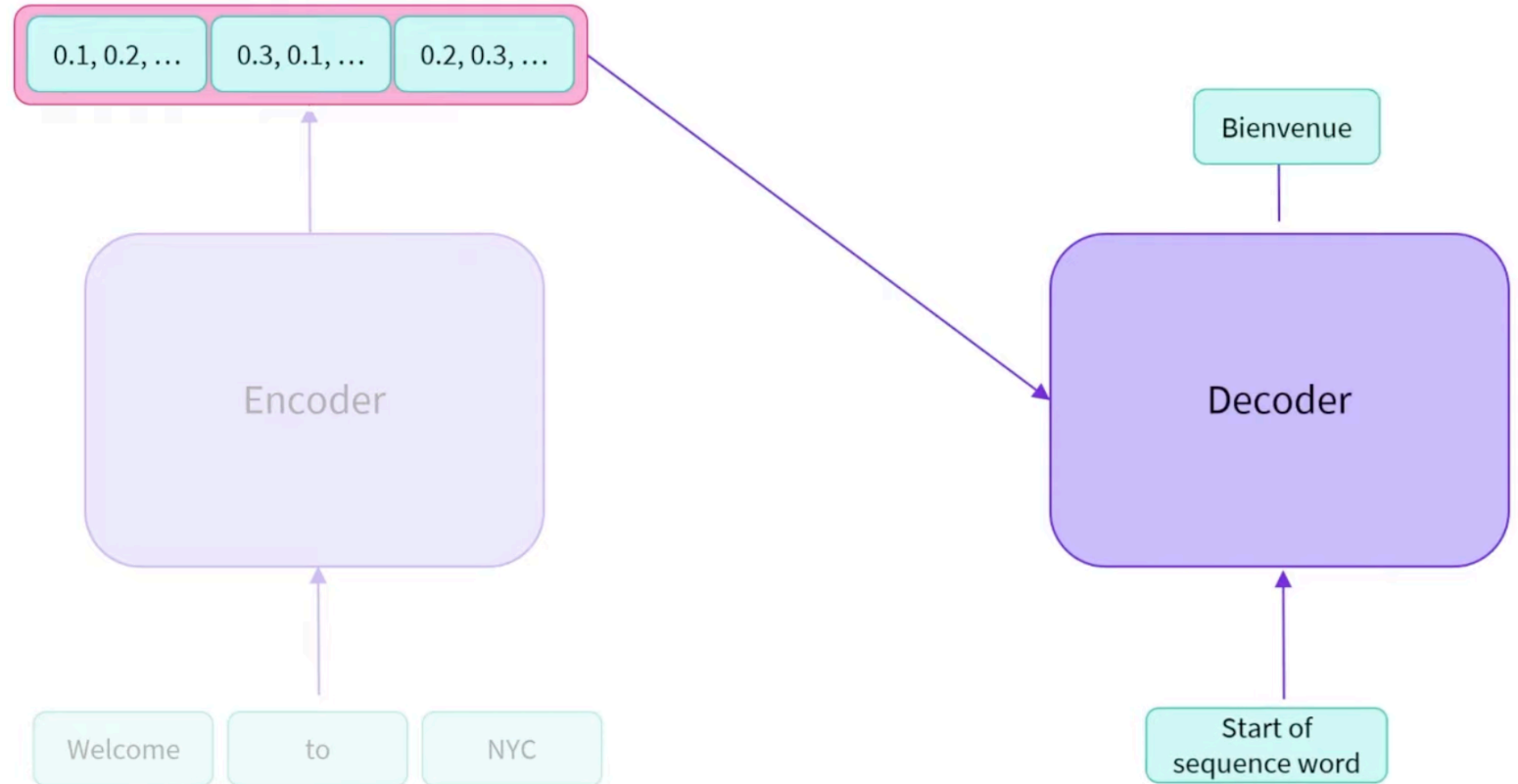
- Encode input sequence

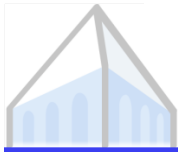




# Encoder-Decoder Inference

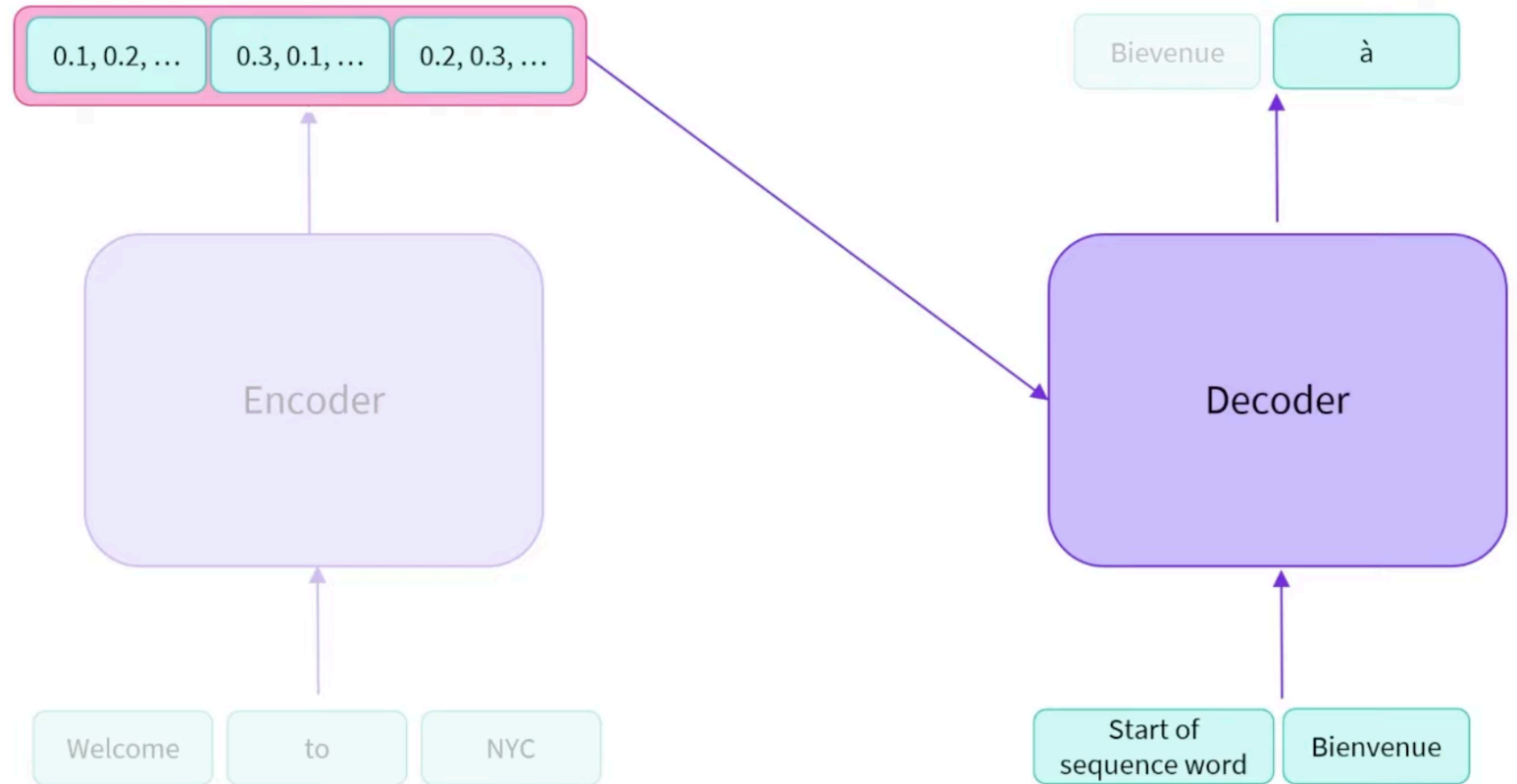
- Encode input sequence
- Attention over input token representations and <start>

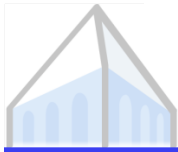




# Encoder-Decoder Inference

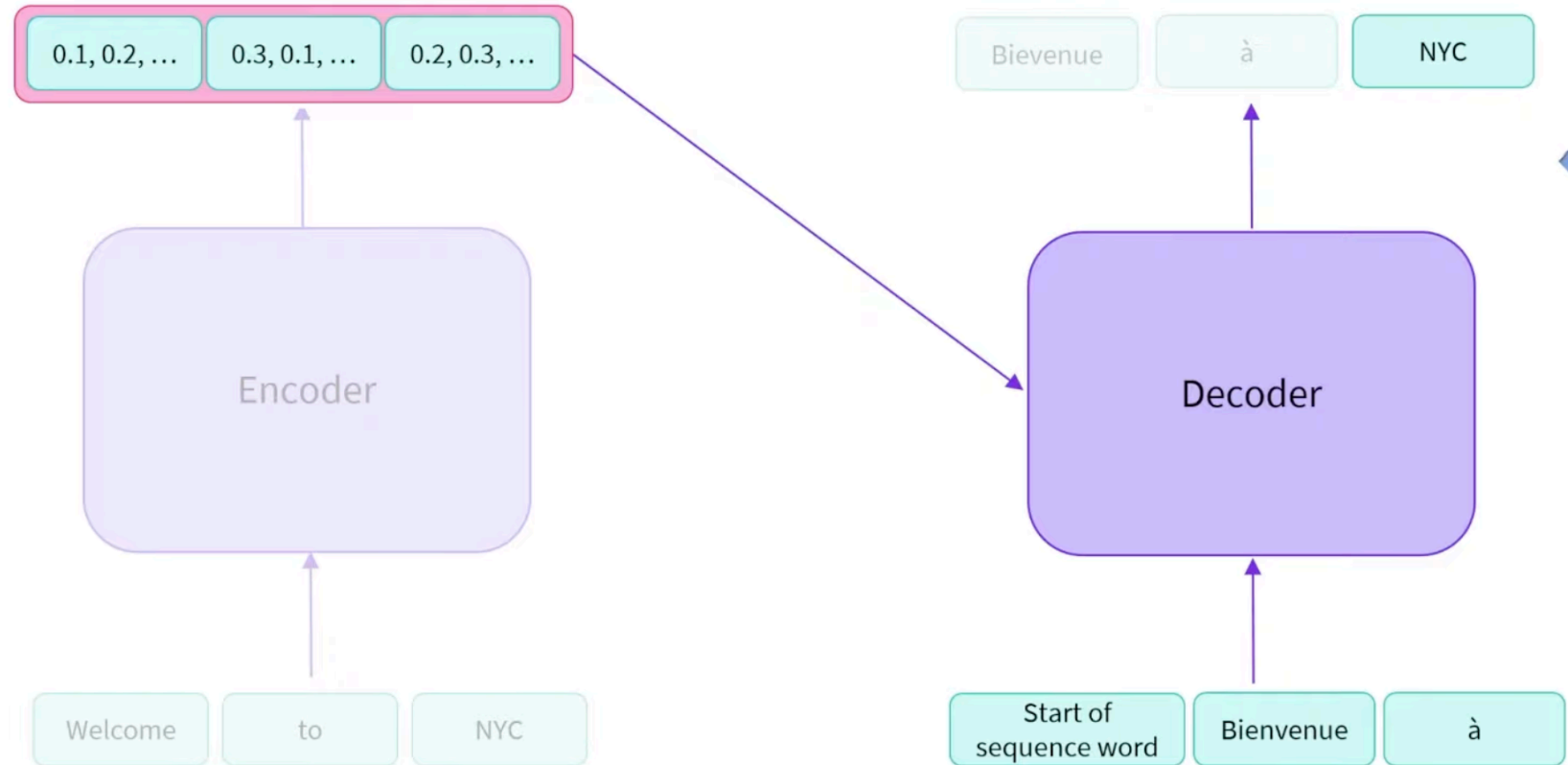
- Encode input sequence
- Attention over input token representations and <start>
- Self-attention

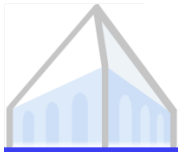




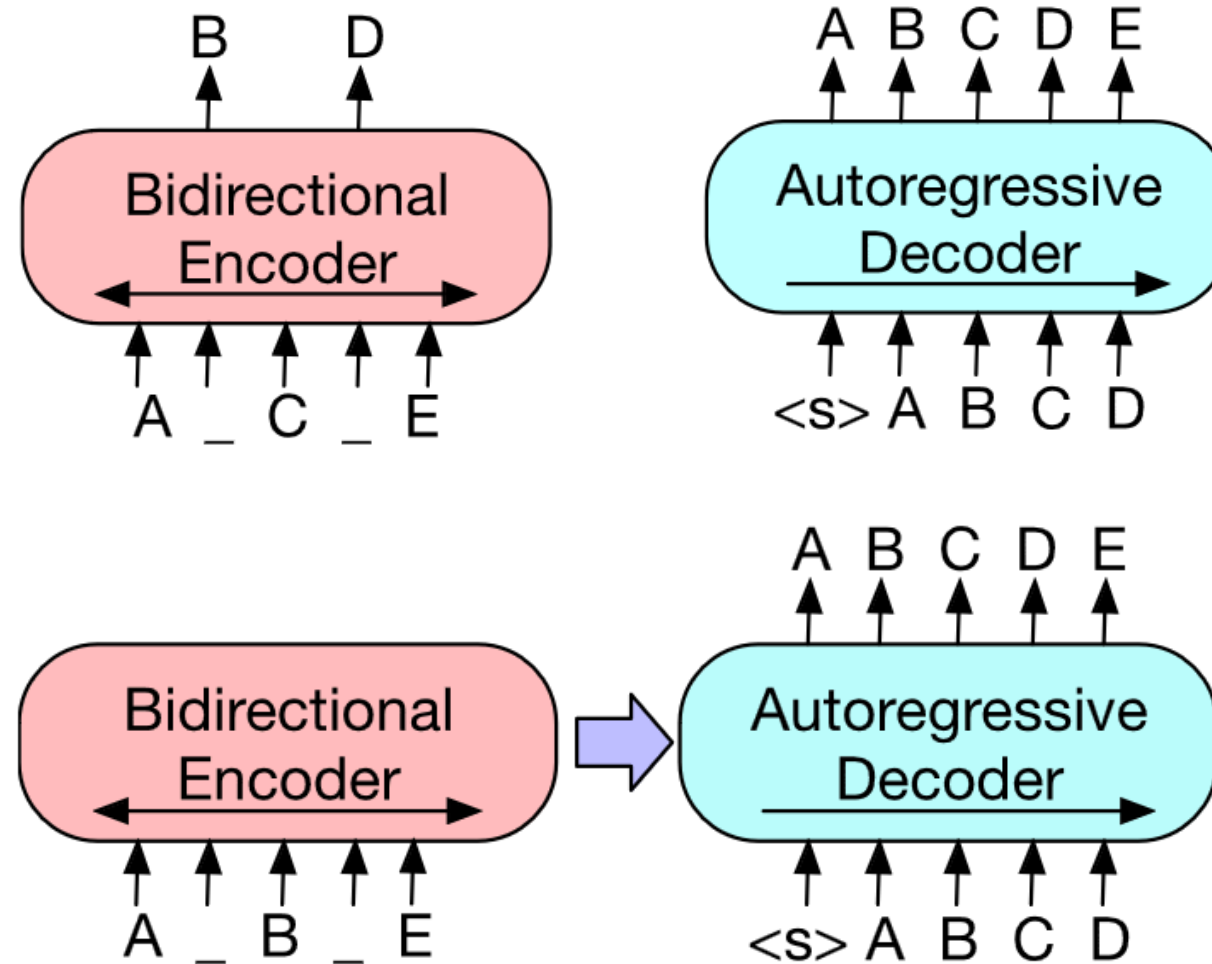
# Encoder-Decoder Inference

- Encode input sequence
- Attention over input token representations and <start>
- Self-attention





# Encoder, Decoder, Encoder-Decoder



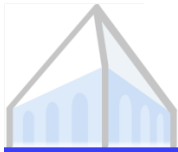


# Problems with the Transformer?

---

- Fixed context lengths “solved” with position embeddings
- Self-attention has quadratic cost  $O(n^2d)$
- Plug: Annotated Transformer (Sasha Rush):  
<http://nlp.seas.harvard.edu/annotated-transformer/>

# Training Language Models



# Recap: Language Modeling Objective

---

- Assume we have training data  $\langle x_0 \dots x_T \rangle$
- Use current LM parameters to compute probability distributions over each token independently, conditioned on the prefix:

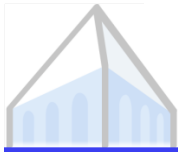
$$P(X_i) = p(\cdot \mid \langle x_0 \dots x_{i-1} \rangle; \theta)$$

- Loss for step  $i$  is cross-entropy between true distribution  $p^*$  (i.e., one-hot) and predicted distribution:

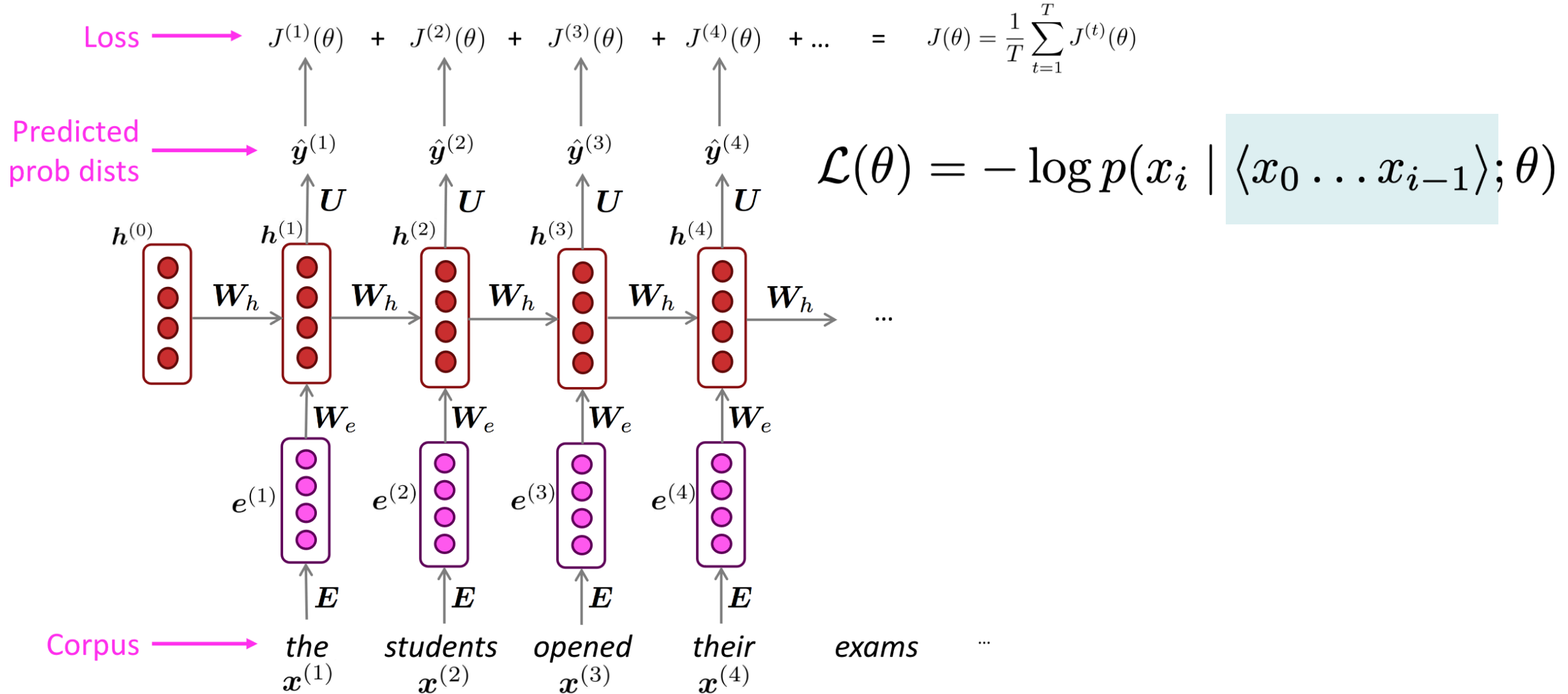
$$\mathcal{L}(\theta) = - \sum_{x \in \mathcal{V}} p^*(x_i = x \mid \langle x_0 \dots x_{i-1} \rangle) \log p(x_i = x \mid \langle x_0 \dots x_{i-1} \rangle; \theta)$$

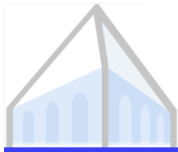
$$\mathcal{L}(\theta) = - \log p(x_i \mid \langle x_0 \dots x_{i-1} \rangle; \theta)$$



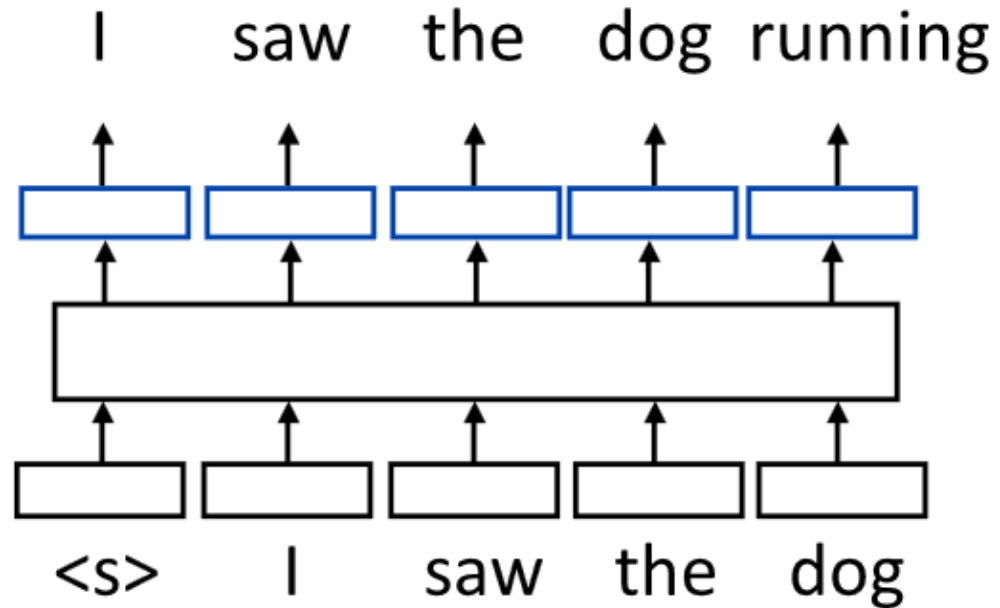


# Next token prediction



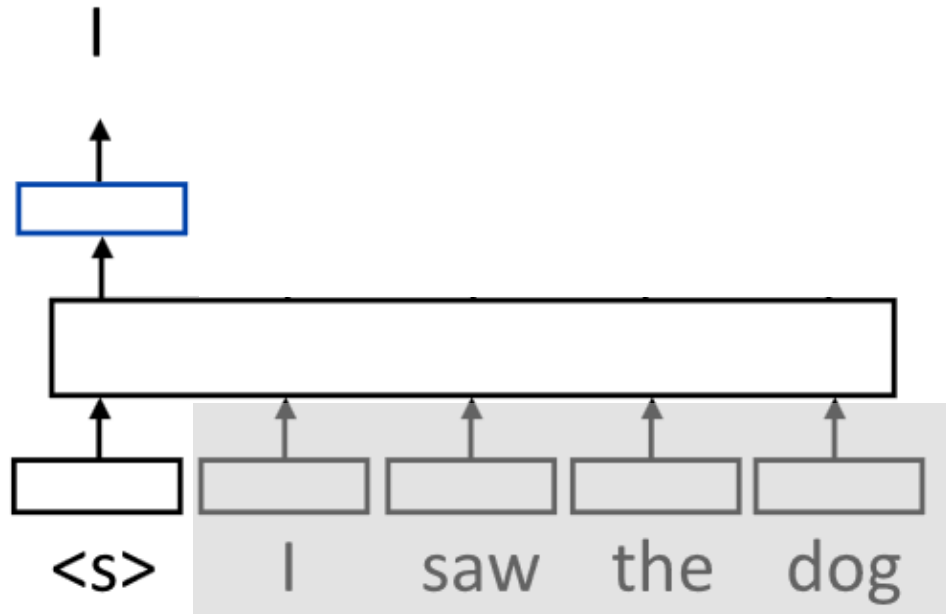


# Next token prediction in Transformers



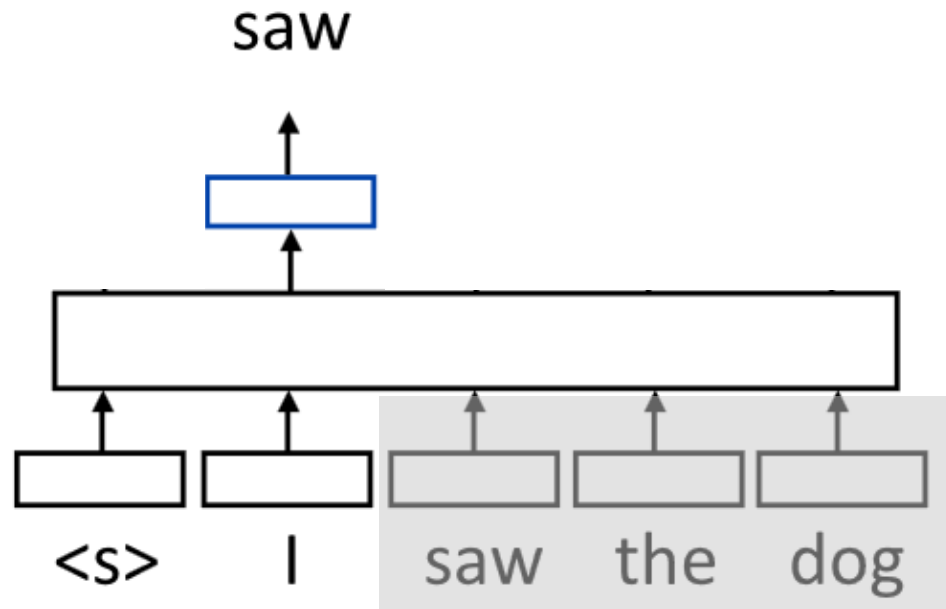


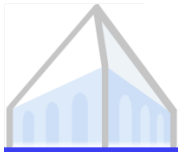
# Next token prediction in Transformers



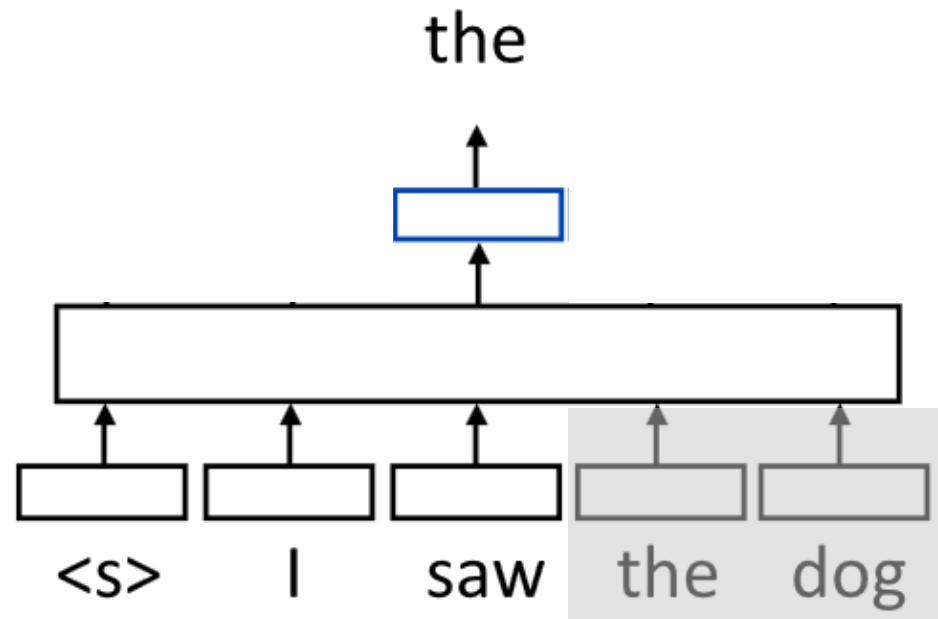


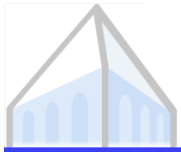
# Next token prediction in Transformers



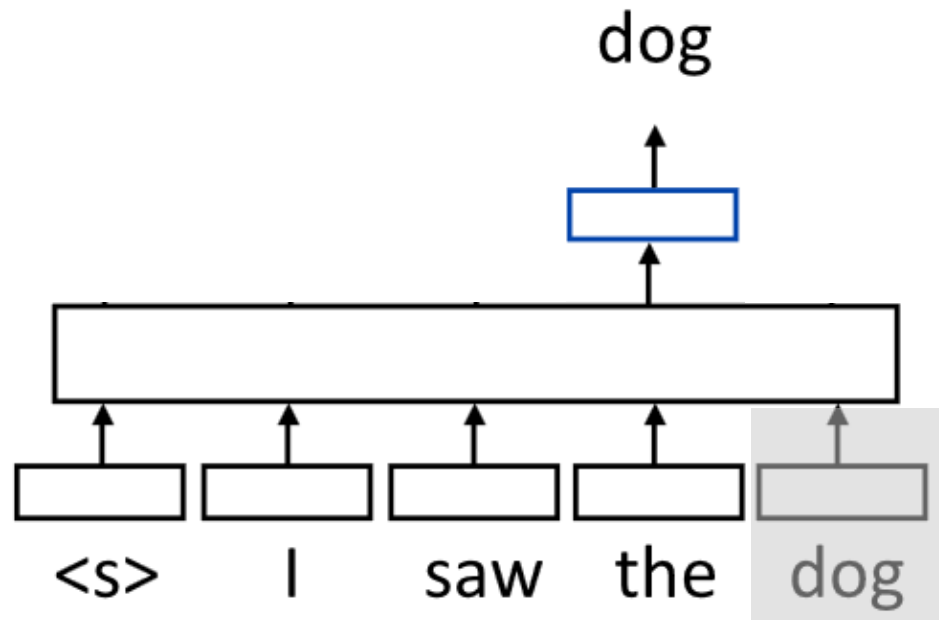


# Next token prediction in Transformers



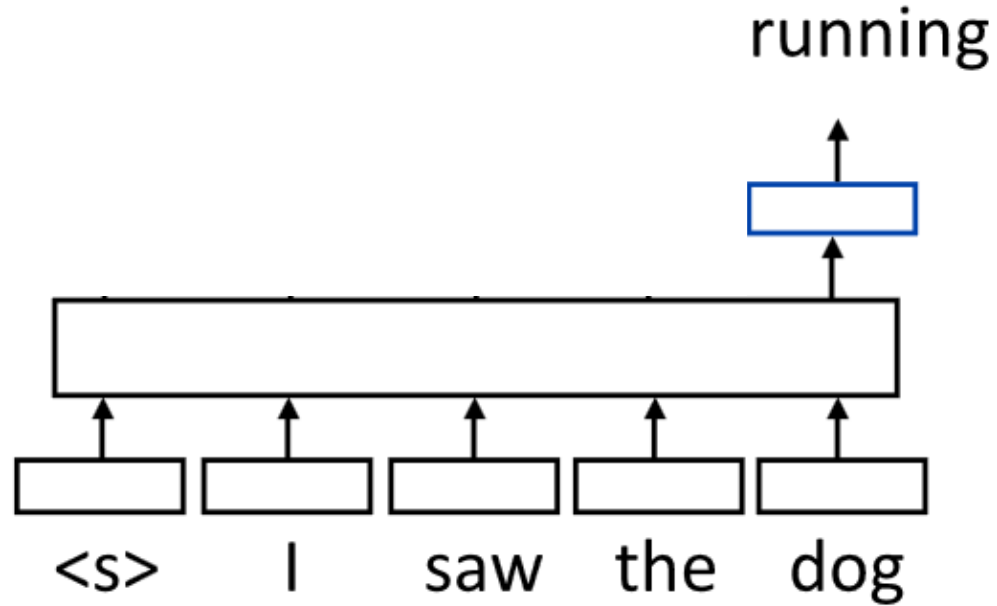


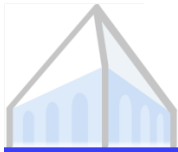
# Next token prediction in Transformers





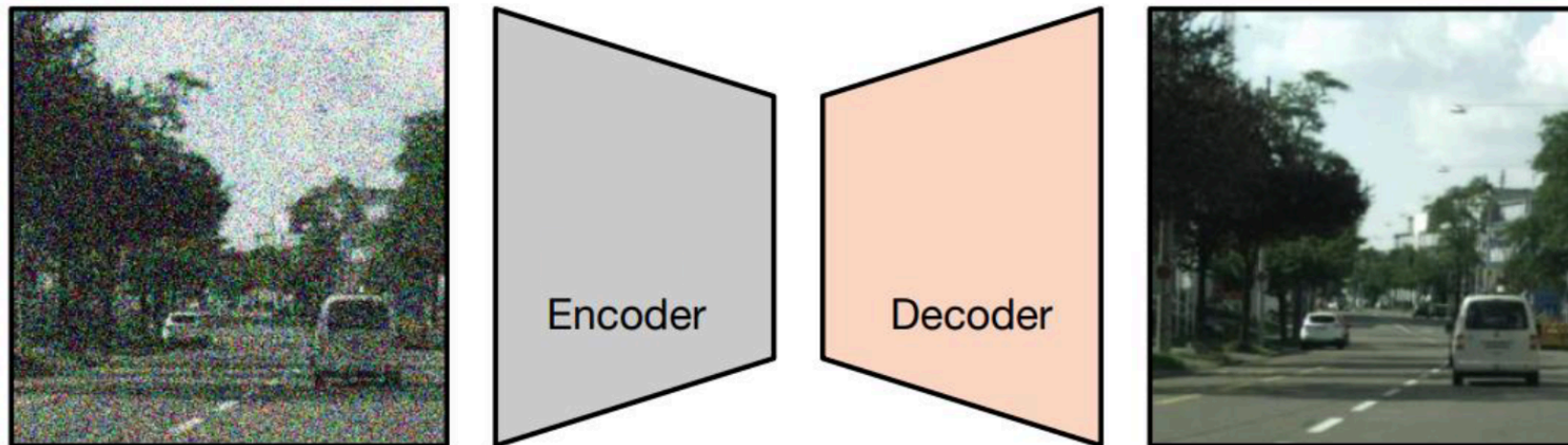
# Next token prediction in Transformers



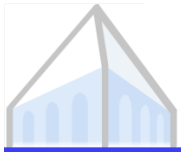


# Denoising Objectives

- Our goal: learn a distribution over text sequences
- Our assumption so far: this distribution is only backwards-looking (conditioned on prefix of the sequence)
- What if we remove this assumption?

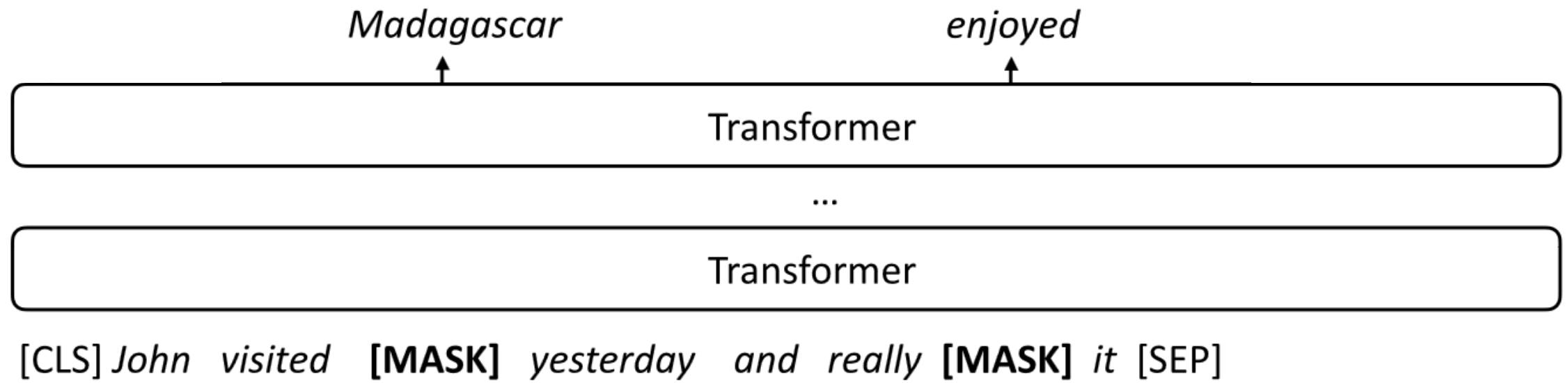


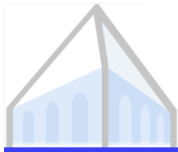




# Masking / Infilling Objectives

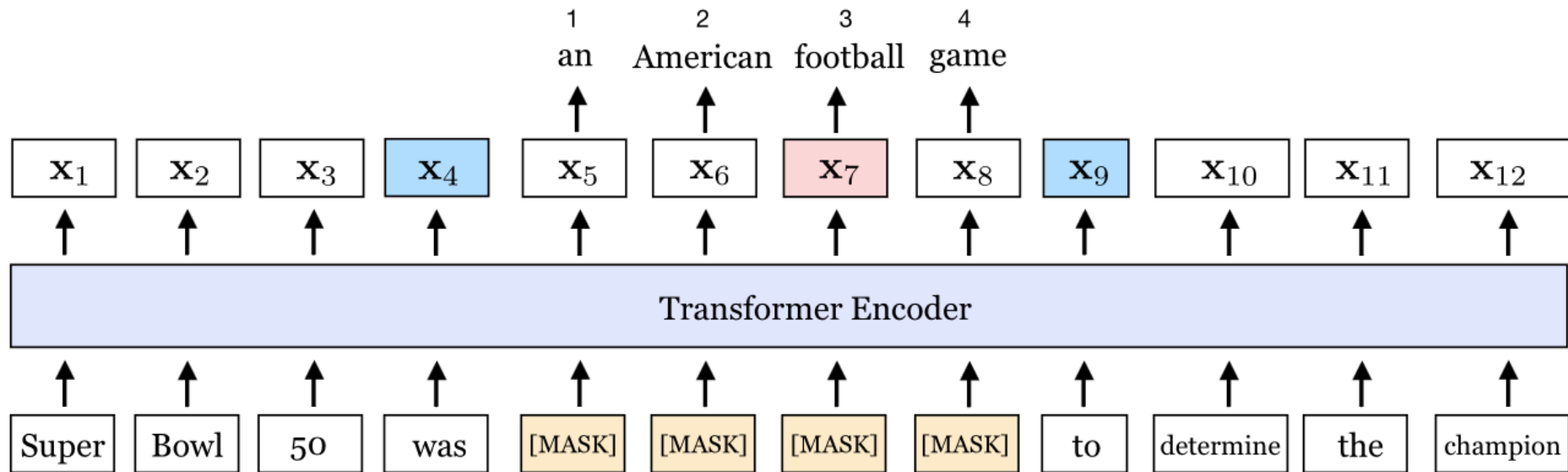
- Randomly mask out ~15% of tokens in the input, and try to predict them from past *and future* context

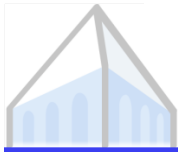




# Masking / Infilling Objectives

- Randomly mask out ~15% of tokens in the input, and try to predict them from past *and future* context
- Or mask out spans of text





# Auxiliary Objectives

